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80,000 Children's Reactions to Meaning in Arithmetic
C. NEWTON STOKES

Certain Ability Factors and Their Effect on Arithmetic Achievement
LELAND H. ERICKSON

An Experimental Study in Teaching Percentage
RUSSELL A. KENNEY AND JESSE D. STOCKTON

The Role of Attitude in Learning Arithmetic
J. PETER FEDON

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Table of Contents

THE ARITHMETIC TEACHER

Volume V Number 6

December 1958

80,000 Children's Reactions to Meanings in Arithmetic*

C. Newton Stokes
Temple University, Philadelphia, Pa.

We have been faced with the problem of improving learning in arithmetic for a long time. Many of us have given considerable attention to its solution. For the past quarter of a century the consensus among us has been that the meanings approach in teaching and learning was the means of a solution.

The brief report that I have to present here summarizes some of the experiences that a rather sizeable group had in an attempt to determine the effects of the meanings method. This group proceeded from the point of view that there should be careful consideration of *What We Should Teach* and *How We Should Teach*.

Accordingly a group of 566 teachers agreed to collaborate with me to see what we could find out. (All of these persons had been students in my graduate courses in Curriculum and Methods in Arithmetic at Temple University.) The length of the study that was organized and carried out covered the 15-year period from the fall of 1940 through the spring of 1955. It was divided into two parts, namely 1940–50 and 1950–55. In the first period we set ourselves to the task of finding the best organization of the What and the How. Then in the second we chose to check every detail of the judgments made at the end of the first period.

Now, at the very outset of the study, we adopted the slogan, "Know What We Are About." In this it was agreed that we must have a philosophy that prescribed our purposes in educating boys and girls. Also there must be an understanding of the contribution that the learning of arithmetic can make in the total process. Accordingly the primary assumption that we educate for the purpose of helping the learner to get on in the world was accepted. More pointedly it was stated as "equipping the individual with a body of understandings through the use of which he would have control of his behaviors in his living in a complex social order." We like to think that we had the same concept as that of the eminent mathematician and philosopher Cassius J. Keyser when he wrote in the early 30's that education has for its purpose the "development of those abilities in the individual which would qualify him to find or create a good and beautiful existence for himself."

Consequently, by any use of our intellect, an analysis of the implications of our postulated purpose would prescribe that our goal was the thinking personality. Or perhaps

* An address delivered at the 36th Annual Meeting of the National Council of Teachers of Mathematics at Cleveland, Ohio.

just as aptly said, we must produce the individual that acts upon thinking. He must be trained so that he will know what to do rather than be taught what to do.

Now it was our belief that no other subject in the curriculum can offer more in producing what we want than can arithmetic. We firmly believed that the mastery of techniques of straight thinking, found through the study of arithmetic, will qualify the learner to act on thinking in the affairs of daily living. Through training in the thought processes found in arithmetic the individual is seen to emerge as a disciplined entity, being able to coordinate intelligently his impulses, habits, understandings and skills in the achievement of his own goals. He becomes an emancipated performer, free from biases, fanaticisms, prejudices, jealousies, and the like. He achieves magnanimity—largeness of purpose, initiative, industry and autonomy—with a will to achieve worthwhileness in every behavior.

Program Determinators

With such a goal in mind it was clear that we could not obtain help in the solution of our problems if we tried to adapt the status-quo determined program (wherein the inflexible traditional action predominates). Nor could we accept the circumstances determined program (wherein courses of study, textbooks, or even pressure groups dominate the prescribements). It was acknowledged that these had been in operation in many places and the outcomes showed that no improvements had been made. We therefore were obliged to create something new. We called our formulation the Principles Determinator, based upon the child's needs and interests, and upon how he learns.

Seeing life and learning to live requires a set of sociological objectives as directives. We therefore had to go to the children themselves for bases from which our principles of procedures were to be determined. From them we could find the *What* and the *How* as they went about the business of making their adaptations to the complexities of the social environment. We could find what their problems were and how they went about solving them. So, within the first 10-year period, 350 of the cooperating teachers proposed to find what children's problems of living were and how the arithmetic in them should be taught so that the relations in them could be most readily comprehended. For the problems, each teacher followed each of her pupils outside of the arithmetic class period for a total of 12 hours. She kept a record of the social problems met and the arithmetic needed for their solution. She also obtained the cooperation of the parents of her pupils. They kept journals of similar data weekly, problems in and about home living. Case studies of upwards of 72,000 children, residing in 12 different states, were thus obtained. It was the materials in these studies which provided us with a curriculum, grade by grade from I through VIII. We defined the minimum essentials as those problems that were met by 60% of the cases at each grade level. For a group composed of 25% of the cases, who had not met all of the problems, the same minimum essentials were prescribed. Then for the remaining 15%, whose lives were richer and fuller, who had encountered problems not faced by the other 85%, we had materials that were usable for enrichment.

Nature of the minimum content

The analysis of the problems of the 60% group gave us the following content in whole numbers. (If space would permit we would present similar materials for fractions, decimals, percentage and geometric relations. We refer you to Scope and Sequence of Concepts in any of the Teachers' Editions of *Arithmetic in My World*, Allyn and Bacon, Inc., Publishers.)

Grade I. Addition, subtraction and multiplication facts in numbers 1 through 9.
Grade II. Addition, subtraction and multiplication facts in numbers 10 through 19.
Grade III. Study of numbers 20 through 39, involving facts and processes.
Grade IV. Study of numbers 40 through 99, involving facts and processes.
Grade V. Study of numbers in hundreds, involving processes.

Grade VI. Study of larger numbers, involving processes.
Grade VII. Applications in areas involving personal affairs.
Grade VIII. Applications in family affairs and in the internal and external properties in geometrical forms.

Organization of Content

The next task to be faced was the organization of the problems into a sequence that was workable both psychologically and pedagogically. Whatever we teach must be learnable, and we must teach in a manner so that the objectives in the social and arithmetical realms of the growth process are certain to be realized. In finding some of the answers in the solution of this problem, the analysis of the children's needs and interests gave us direction. The information that was collected on the out-of-arithmetic classroom situations helped us to identify ten areas of social growth needed by every learner. Also the nature of these problem situations helped us to define the stages in learner's maturation process. It was clear therefore that, whatever the organization, there must be an integration of factors which would produce the desired growth in the social areas and the desired progress in moving up through the stages of development.

The problems defining the different areas of growth were classifiable according to the following categories,

1. Providing one's self with food, shelter and clothing
2. Giving attention to physical growth and physical fitness for work and play
3. Recognition of desired relations with individuals, the family group, and the social group
4. Improving self as a personality (one's own individuality)
5. Exercising health routines as age increases
6. Attention to personal grooming
7. Responsibility for contributions to society
8. Developing integrity in one's own conduct and respect for authority
9. Ascertaining the methods of controlling the social and physical environment
10. Learning how to communicate ideas to others.

Now, in order to organize with respect to levels of maturation we observed that we must give attention to the specific stages which we characterized by the terms intimacy, identity, initiative, industry and autonomy. In the intimacy stage we saw the child as an imitator through the first grade and on into the first few weeks of the second grade. Then all at once we saw him change. He belonged in the identity stage where he recognized that he was an individual. He operated at this stage until about the time that he finished the third grade. Then he had initiative during the fourth grade, industry until about the middle of the sixth grade, and finally automony operated on through the eighth grade.

In terms of these criteria we found the most desirable organization of problems to be that of the *unit plan*, in which a unit is a body of materials that belongs together from the standpoints of logical sequence, relatedness to a theme in social growth, ease in teaching, self-motivating and ease in learning. Through this plan the learner can see purpose in the unified whole; the problems in their relatedness give a story that makes sense and the worth of the learning is seen. Such an organization provides readiness for learning, conceptual developments by easy steps, and drill that is necessary for the production of insight and for the maintenance of skill.

Preparation of Materials

Since there were no published materials for classroom use, we had to prepare mimeographed forms for the teachers. This was done during the summer sessions when some of the teachers were in attendance at Temple University. Year by year we would revise the forms from the previous year, making adjustments where improvements were found, as former ones contained apparent deficiencies.

These mimeographed forms contained details of the units for the year's work, illustrative developments, illustrative devices and suggestive pictures (in sketched form) for motivation. The teachers could take these and prepare lesson plans, story problems, and abstract drill exercises so that the children's experiences in the classroom were complete.

Technics

Effectiveness through the teaching action comes only through the knowledge of how the child learns. All procedures must be adapted according to the principles involved. The 350 teachers learned a great many things about efficient procedures during the first 10-year study. It is probable that one of the most significant was that *children are learning when thinking is in operation.* Another significant finding was that we all had been missing some of the steps necessary for the comprehension of the generalization in most every conceptual learning. It was that we had been making jumps from the observed relationships expressed through the use of manipulative materials to the generalized formulation. When this was the case they found that the reaction of the learner was indeed a rote one, hence no meanings had been comprehended. The problem was to find those steps, from the concrete to the abstract, which took the learner by a gradual process into sound thinking in symbolisms.

This omission of steps can be illustrated as follows:

PROBLEM: How may lengths of the $\frac{2}{3}$ yd. size may be obtained from a piece of rope 6 yd. long?

a. Generalizing too quickly:
6 yd. $\div \frac{2}{3}$ yd. $=$? In the concrete we have

There are three $\frac{1}{3}$ yd. in 1 yd. So, in 6 yd. there are 6 times as many as in 1 yd., or 6 yd. $\div \frac{1}{3}$ yd. $= 6 \times 3 = 18$ lengths of the $\frac{1}{3}$ yd. size. But our divisor was $\frac{2}{3}$ yd. or twice as great as $\frac{1}{3}$ yd. Thus 18 must be divided by 2. Accordingly we have

$$6 \div 2/3 = \frac{6 \times 3}{2} = 6 \times 3/2 = 9.$$

There are 9 lengths of $\frac{2}{3}$ yd. The divisor is seen to be inverted and the computation done by multiplication. The learner however sees no meaning in the value $\frac{3}{2}$.

b. Generalizing through meaningful steps:
Step 1.

By the counting process it is found that there are 9 lengths of the $\frac{2}{3}$ yd. size in 6 yd.

Step 2. A second examination of the concrete shows that there are one and one-half $\frac{2}{3}$ yd. lengths in 1 yd. So, in finding how many there are in 6 yd., we multiply $6 \times 1\frac{1}{2}$. $6 \times 1\frac{1}{2} = 9$. Of course if addition is desired, $1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}=9$.

Step 3.

$$2/3\overline{)1 + 1 + 1 + 1 + 1 + 1} =$$

$$2/3\overline{)3/3 + 3/3 + 3/3 + 3/3 + 3/3 + 3/3} =$$

$$2/3\overline{)\overset{1\frac{1}{2}}{3/3} + \overset{1\frac{1}{2}}{3/3} + \overset{1\frac{1}{2}}{3/3} + \overset{1\frac{1}{2}}{3/3} + \overset{1\frac{1}{2}}{3/3} + \overset{1\frac{1}{2}}{3/3}} = 9$$

wherein the division of $\frac{3}{3}$ by $\frac{2}{3}$ is done rationally—3 units divided by 2 units of like size (L.C.D.) gives on whole length and half of another one.

Step 4. 6 yd. $\div \frac{2}{3}$ yd. $= 6 \times 1\frac{1}{2} = 6 \times \frac{3}{2} = 9$ lengths. Thus the meaning of the value $\frac{3}{2}$ is comprehended in the generalization that involves the multiplication of the inverted divisor.

Other illustrations could be shown as in the process of addition by bridging and endings, multiplication of fractions, finding how many 8's there are in 56, or in fact in the development of any concept.

The reactions of the pupils gave us insight into two significant principles in the learning process. In one we observed that comprehension of symbolic representations is the result of mental action initiated at first through the differentiations of sensory fields and then passing by logical steps into meanings involving the symbolisms. However it is economical to use the sensory only to the extent that suggestions in the use of the symbols become the natural consequence. For some pupils this means that there is need only for a minimum of the sensory or perhaps only for the perceptual. These are those individuals whose maturation qualifies them to excel.

For the other principle we observed that interdependent relations and processes are learned most easily when presented simultaneously. This gave us what we termed a cone of experience method. That is, it offers the opportunity for a pupil to proceed to a sequential piece of learning material even though his comprehension of the prerequisite is at a step below that of a generalized outcome (for example, note how the learner could divide fractions in the above illustration before acquiring the inversion rule). This is a most desirable procedure in guaranteeing forward progress because there is successful performance at the pupil's own level of operation. This cone of experience method allows the learner the opportunity of reaching a higher level of understanding or to operate at a higher level as relations reappear in new forms. Thus all children with the required mental equipment will eventually reach the generalizations. Furthermore it allows the pupil who excels to receive enrichment work while other pupils' levels are being raised. (Enrichment may be the structurization of concepts in different ways, or it may be scrapbook work showing applications in new problem situations.)

The Controlled Experiment

As the teachers worked year by year in the 10-year period of exploration it was apparent that arithmetic learning was being improved. Those schools committed to a standardized testing program were finding exceedingly encouraging results. Thus when it came time to launch the second part of our study we had a large body of directives. We chose 36 new collaborators for each of the first six grades who were commissioned to carry our "cone of experience" method in meanings forward for the period 1950–1955. Their pupils (the experimental groups) were matched with others in the same school systems (the control groups) who were following the standard textbook procedures. The items for matching were,

a. Both groups studied textbook procedures the previous year
b. Matched pairs had I.Q.'s within 5 points, chronological ages within 6 months, Metropolitan pretest scores within 3 months
c. Teachers for both groups in a school system were adjudged equally effective by the school administration

Now two types of measures were used in securing objective data by which an evaluation of our program could be made. We determined the amount of growth for both the control and the experimental groups by finding the differences between the scores on the pretest and a second administration of the Metropolitan Standard Tests at the end of the year's study. The forms of the test were Primary I Battery for Grade I, Primary II Battery for grade II, Elementary Arithmetic Test for Grades III and IV, and the Intermediate Airthmetic Test for Grades V and VI. (We were able to secure 2,000 matched pairs at each grade level.)

We also determined the per cent of sustained attention shown by the pupils during the arithmetic class period. This was determined by means of having an observer hold a stopwatch on pupils during the class period. We carried this measure forward until we had 100 pupils at each grade level, about equally divided among slow, average and fast pupils.

Our findings are shown in the chart below.

DIFFERENCE IN YEARS OF GROWTH OF 2,000 PUPILS IN GRADES I–VI FROM TWO METHODS OF PROCEDURE

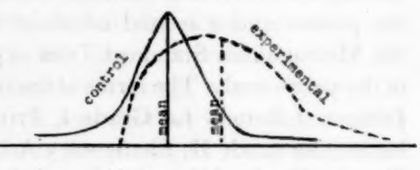
Grade	Control Group	Experimental Group	Difference
I	0.9	1.8	0.9
II	1.0	2.1	1.1
III	1.0	2.4	1.4
IV	1.1	2.1	1.0
V	1.0	1.9	0.9
VI	0.9	2.0	1.1

These data were obtained through actual arithmetic means in both fundamentals and problem solving, thus representing levels of arithmetic achievement. When the differences were treated for reliability the probability on the average that they were true differences was 990 out of 1,000. Consequently,

everything being equal, it appears certain that the meanings (cone-of-experience) method produced results that were, on the average, one year above those under the control method.

Suppose now that we examine a typical sketch of the achievement scores at one grade level. Placing both the control and the experimental groups on the same base line, we have the following:

GRADE III



It is to be noted from the sketch that the low-score tail of the curve for the experimental group is missing. This indicates that meaningful teaching by the cone-of-experience method gave every learner a level of successful operation.

A finding not presented in the table given above is the growth of Grade I children for the first half-year of study. Here it was determined that the control group that initiated number study through rote counting were one and one-half months behind the experimental group which had its beginning work as the recognition of group values without counting.

Now, for the second measure, that of sustained attention, it was found that the experimental groups gave us an average of 92% concentration while the control groups had an average of 66%. These figures were obtained when the length of class periods in both groups were as follows:

Grade I, 15 minutes; Grade II, 20 minutes; Grade III, 25 minutes; Grade IV, 30 minutes; Grade V, 35 minutes; Grade VI, 40 minutes.

We checked also on the extension of these periods for several minutes, and too, if there were variations when the class met in the morning or the afternoon. For the former there was a distinct let-down in attention in the extra time (approximately 15%) while for the latter there were no distinct differences. Thus it appears that we have found the appropriate class-period lengths and that the time of day for the arithmetic study is of no concern.

Finally, our cone-of-experience method in step by step conceptual developments, offering enrichment for those who excel, made it possible for pupils to complete from four to twelve supplementary projects within the year at each grade level. The nature of these experiences called for the application of comprehended meanings rather than acceleration in the subject of arithmetic. The children in the controlled groups had time for none of these enrichments.

Summary

Now, in summary, it appears that we have evidence that a program of meanings will improve learning in arithmetic. To support this contention we see every pupil successfully operating at his own level with achievement, on the average, as measured by the Metropolitan tests, one year higher than that of pupils studying standard textbooks of the time, and with motivation producing 92% sustained attention as compared with 66% when the program is determined by these same texts, and with enrichment a reality for the pupils who excel.

(Finances involving the need for the standard tests were supplied by Temple University, Philadelphia, Pennsylvania.)

EDITOR'S NOTE. Dr. Stokes has given us a brief summary of his long and large experiment which developed a method and procedure for teaching and learning arithmetic with understanding and for which he gives data in comparison with a control group representing six grades for one year. Assuming other factors being equal, the experimental group surpassed the control group by one year of achievement as measured by a standardized test. One wonders if Dr. Stokes had constructed a good test of arithmetical concepts and principles which would measure understandings, if the results might not be more pronounced. The editor liked the statement of aim in arithmetic teaching "... produce the individual that acts upon thinking." This is very different from and at a much higher level than learning to compute with numbers. With adults, it is easy to detect the non-thinking person and the one who may think but does not have the compulsion to act upon his thinking. It would appear that Professor Stokes and his associates have established a good case for teaching a "thinking arithmetic" founded upon understanding.

Certain Ability Factors and Their Effect on Arithmetic Achievement

LELAND H. ERICKSON
Franklin College, Franklin, Ind.

A BRIEF REVIEW of the research which has been accomplished during the past thirty years in elementary arithmetic reveals several centers of interest. It can be generally stated that a large proportion of the research is concerned with teaching methods, patterns of thinking in dealing with various arithmetic concepts and operational skills and research which is concerned primarily with the arithmetic curriculum.

An examination of the profound changes which have been made in textbook materials and teaching techniques serves to reveal the influence which research has had upon the experiences which children have in elementary arithmetic. The importance of research cannot, however, be entirely measured in terms of observable changes. For example, it might reveal a better way of teaching, or on the other hand, it might throw some light upon a problem not previously considered as being important. Research may verify or give substantial evidence to something already known. On the other hand it could, and often does, shed some light which tends to refute a previously acquired false notion.

This paper is primarily concerned with focusing attention upon the learner; specifically, effort has been made to examine statistically certain ability factors and how they effect arithmetic achievement. The data which are presented in this article are primarily concerned with the following:

(1) The relationship of intelligence and achievement in arithmetic;
(2) Relationships between intelligence, arithmetic concepts and problem solving;
(3) Relationship of reading ability to arithmetic achievement;
(4) Relationship of socio-economic status and arithmetic achievement.

It is important to mention at this point that for most cases under consideration statistics have been computed, not only for the total sampling, but also to show certain differences between various levels of arithmetic achievement, specifically, the upper 27 per cent, middle 46 per cent and the lower 27 per cent. This is important because without this comparison several significant inferences would be lost. In fact, as will be revealed later, it is highly important that researchers give more consideration to various achievement and ability levels in suggesting methods of teaching, curricular modifications and materials. It is believed by the writer that some of the evidence presented in this paper, because of its objectivity and statistical analysis, will fall under one, or in some cases several, of the categories mentioned earlier. There are certain implications revealed in this study which, of course, need further study and verification.

This study involved 269 sixth grade pupils in eight classrooms of four public elementary schools located in a midwestern city school system. The schools used in the study were selected primarily on the basis of socio-economic locations. The only criteria which were applied to make the selections of the schools used in the study were opinions generally held by teachers and administrators. Two schools are largely made up of pupils considered to be from homes of relatively high socio-economic culture, while the

other two schools consist of pupils who for the most part are from homes of a somewhat lower socio-economic culture. It was felt that data gathered from these sources would offer certain research possibilities not available from unselected resources. The instruments used to obtain the data consisted of the sixth grade arithmetic section of the Iowa Tests of Basic Skills, Form 2, 1956, the Otis-Quick Scoring Mental Ability Test, the Iowa Silent Reading Test and a series of questions carefully prepared and administered for the purpose of gaining certain information not available from other sources.

Intelligence and Arithmetic Achievement

No one can successfully deny the importance of intelligence in learning anything requiring thought. There are, however, some necessary cautions which should be taken in interpreting too loosely certain statistical evidence concerning intelligence and achievement. For example, as can be seen from Table I, the Correlation Coefficient of 0.72 shows a relatively high relationship between I.Q.s and arithmetic sources when only the entire sampling is considered. However, the breakdown into arithmetic achievement sub-groups shows a considerably lower relationship for each. The difference in magnitudes between the correlation coefficient of the entire sampling and the coefficients of the sub-groups may, in part, be due to the skewed distributions in the sub-groups. This is usually the case when relationships within sub-groups are compared to the entire sampling. However, the relationship between intelligence and arithmetic achievement within each sub-group is lower than is generally assumed. For instructional use the data which are shown in Table II presents a clearer picture of the levels of achievement and accompanying distributions of intelligence. As can be seen from Table II, children with average and above average I.Q.s as well as children with lower I.Q.s are represented in the lower 27 per cent, while a number of children with average I.Q.s and lower than average I.Q.s are with the higher I.Q.s represented in the upper 27 per cent. The writer feels that implications shown here are important. For example, in one instance several years ago it was known that a teacher of general mathematics made a statement to the effect that students having academic difficulty in his classes were of sub-normal intelligence. This statement was somewhat falsely supported by saying that low arithmetic achievement went hand in hand with low intelligence. As you can see in Table II this is not always the case. Further investigation revealed that this teacher's impression, as far as his classes were concerned, was arrived at without knowledge of their mental abilities, but rather judgment of intelligence was based entirely upon achievement in mathematics. No

TABLE I

RELATIONSHIP BETWEEN INTELLIGENCE AND GENERAL ARITHMETIC ACHIEVEMENT

Group in Arithmetic	Number of Pupils	I.Q. Range	Correlations: Intelligence Quotient and Total Arithmetic Score
Entire Sample	230	54–137	0.72
Upper 27 per cent	63	84–137	0.39
Middle 46 per cent	104	75–130	0.46
Lower 27 per cent	63	54–113	0.40

TABLE II

LEVELS OF ARITHMETIC ACHIEVEMENT AND ACCOMPANYING DISTRIBUTIONS OF INTELLIGENCE

Intelligence Quotient	Number of Pupils in Arithmetic: Lower 27 Per Cent	Middle 46 Per Cent	Upper 27 Per Cent
50–59	1		
60–69	4		
70–79	3	1	
80–89	18	7	1
90–99	18	23	2
100–109	14	30	10
110–119	2	30	20
120–129		9	26
130–139			5

doubt this teacher as well as other teachers has seen a statistic such as the 0.72 correlation coefficient in Table I and applied it too broadly. It can readily be seen that such an interpretation could greatly affect teaching methods. Teachers should be aware of both the mental maturity of pupils and their achievement in arithmetic, and I.Q.s should usually be converted into mental ages for most accurate and functional classroom use. An ultimate goal is to strive for each pupil's arithmetic achievement, sometimes converted into arithmetic age, to approximate or surpass his mental age. There are so many factors which enter into the complex pattern of acquiring arithmetic concepts that this goal may seem somewhat idealistic. The factors of ability, attitude, interest, aptitude, previous experiences, reading ability and other factors all enter into the total picture of arithmetic achievement. Nevertheless, efforts on the part of teachers to help children use to the optimum their intellectual potential is a practical goal for which to strive. Arithmetic ability correlates sufficiently high with I.Q. that, with proper interpretation, mental age can be used as a basis of expectation of a pupil's arithmetic ability.

Relationships and Inter-relationships Between Intelligence, Arithmetic Concepts, and Problem Solving

A structure is no better that its foundation. Thoughtful elementary teachers of recent years are mindful of this fact. They are trying to provide elementary children with a sequence of quantitative experiences which will not only meet their present social needs, but which will also give them understanding and insight into more complex concepts and operational skills in the future. If all children were of the same ability and if they all learned in the same ways this would not be a complicated task. Since there are, however, many factors which enter into this very complex developmental process of learning, teachers are seeking further insights into the problems which underlie maximum arithmetic achievement by all children. Some of the factors which have implications for teaching, and a direct bearing on these problems, are brought to the attention of the reader in the paragraphs which follow.

Before commenting upon the relationships involving intelligence, arithmetic concepts, and problem solving, a brief explanation of the terms "arithmetic concept" and "problem solving" is needed. The term, arithmetic concept, as used in this paper is concerned with the understanding pupils have of the number system and the terms and operations used in arithmetic. The term, problem solving, is concerned with the application of these basic concepts and operations to functional problem situations. These situations requiring quantitative operations are sometimes called "story problems" or "thought problems."

It can be seen from the data in Table III that the relationship between intelligence and arithmetic concept scores is relatively high while the corelation between I.Q. and

TABLE III

COMPARISON OF I.Q.S AND ARITHMETIC SCORES

Groups in Arithmetic	No. of Pupils	I.Q. Range	Correlations		
			I.Q. and Total Score	I.Q. and Concept Score	I.Q. and Problem Solving Score
Entire Sampling	230	137–54	0.72	0.73	0.58
Upper 27 per cent	63	137–84	0.39	0.40	0.19*
Middle 46 per cent	104	130–75	0.46	0.47	0.15*
Lower 27 per cent	63	113–54	0.40	0.47	0.05*

* These correlations are no higher than chance.

problem solving is significantly lower. The correlation coefficient of 0.73 describes the relationship between I.Q. and concept scores, while the relationship between I.Q. and problem solving is shown by the considerably lower coefficient of correlation, 0.58. There is a statistically significant difference between these correlations.

Does the high correlation between intelligence and concept score indicate need for more differentiation of classroom experiences for the various ability groups? That is, should methods of presentation, materials used, and time consumed be substantially different for children of different mental ages? The writer believes that in many instances the answers to these questions are in the positive. A reasonable assumption is that children of varying mental ages do, for the most part, gain understandings in different ways and that they need varying amounts of time to learn concepts and practice operational skills. The methods or method of accomplishing this is up to the classroom teacher and the school administration. That is, some teachers and administrators may desire a homogeneous type of grouping within class grades; others may want heterogeneous organization and grouping within rooms. Some teachers may feel that they can handle this problem without any formalized groupings. Perhaps there is no single solution to this problem. It seems very important, however, that administration and faculty together give this problem considerable study and decide upon some specific plan for organizing a developmental program for children of varying abilities. The facts seem clear. Children of varying abilities need different kinds of learning experiences. This is a good starting point in trying to do something specifically to improve arithmetic instruction in the elementary school.

Why is the relationship between I.Q. and problem solving so much lower than the relationship between I.Q. and concepts learned, the correlation coefficient for the former being only 0.58? The implications seem much more important than merely trying to answer an academic question; yet it is difficult, if not impossible, to give an entirely objective answer or answers from the data at hand. At the risk of being frowned upon for making assertions, not entirely proven by objective evidence, the writer will suggest several inferences which might be made.

A factor which enters into the complex process of solving thought problems is that of reading ability. The factor of reading might also contribute to difficulty in applying operational skills and other concepts to nonverbal problems if directions must first be read. Comments regarding the relationship of reading ability to arithmetic achievement will be elaborated upon more fully in another section of this study.

One reason for the significantly lower correlation between I.Q. and problem solving might be for the reason that considerably more emphasis has been given to classroom experiences involving number operations and terms at all ability levels rather than in applying them to thought problems during the first five years of school. In some respects this seems logical. However, the data seem to reveal that children of higher ability probably have spent too much time practicing operational skills and have not had enough opportunities for applying them. Generally speaking, earlier and more frequent opportunities for all levels to apply number operations and concepts to functional problem situations seems desirable.

It is revealing to note that the large majority of pupils, when asked to indicate their preference for performing certain arithmetical procedures, gave working thought problems a very low rank. It seems logical to assume that one reason for the very low ranking is their lack of confidence in applying quantitative procedures to problem siutations.

In some cases pupils learn certain basic number operations largely without understanding, by drill; that they remember how to apply these skills as a result of having used them in similar problems situations time and time again is entirely possible. Children may learn certain arithmetical procedures with-

out meaning, because they were taught in this manner. It should not be overlooked, too, that certain operational procedures and terms are quite meaningless to some pupils when presented in isolation. However, when certain skills and concepts are put to use in solving problems which have meaning to them, the skills which have been applied seem to become more meaningful.

Perhaps the data presented in Table IV will help to throw some additional light on this problem.

TABLE IV

RELATIONSHIP OF PROBLEM SOLVING ABILITY AND KNOWLEDGE OF ARITHMETIC CONCEPTS

Groups in Arithmetic	Number	Correlations: Problem Solving Scores and Arithmetic Concept Scores
Entire Sample	230	0.73
Upper 27 per cent	63	0.22
Middle 46 per cent	104	0.04
Lower 27 per cent	63	−0.24

While the data indicated in Table IV show a reasonably high correlation of 0.73 between problem solving scores and arithmetic concept scores for the entire sample, it is important to note the near zero relationship for the middle 46 per cent and the significantly high negative relationship of −0.24 for the lower sub-group. While the inferences which can be made from the data presented in Table IV are not all conclusive, it does seem apparent that this evidence tends to give some support to statements previously made about I.Q. and problem solving relationships. The need for further studies and experiments to help find out the different ways in which children of all abilities gain insight into a functional understanding and use of quantitative operational skills and concepts seems extremely important. Certain questions might be asked. For example, is too much time being consumed trying to teach children of low ability certain arithmetical understandings and meanings beyond his grasp? Do high achievers spend too much time practicing on number operations and certain arithmetic skills without enough time being given over to problem solving? Are certain methods of teaching arithmetic concepts highly successful in their use with children of relatively high ability in arithmetic, but only frustrating to children of lower ability? The answers to these and similar questions are necessary if teachers are to help all children to learn arithmetic to the maximum of their ability.

Relationship of Reading to Arithmetic Achievement

It should not be overlooked that the ability to read accompanies to a very great extent the ability to solve thought problems which pupils are expected to read. It is not at all unusual for pupils early in the grades to be poor readers yet to be very successful in manipulating number operations and in writing answers to non-verbal problems. Research bears out the fact that poor readers are sometimes very successful in working number problems in which the operation or operations are indicated. However, as pupils proceed through the grades the problem of reading becomes more burdensome to the poor reader in arithmetic. More and more stress is placed upon verbalized problems which must be read, and this load must be added to an arithmetic program already heavy with the new complex concepts. In some instances inability to read is mistaken for inability to apply arithmetic concepts to thought problems. Frequently children with reading difficulties have vocabulary and other language deficiences which affect their problem solving abilities.

Table V shows the relationships between the mathematics grade equivalent, reading vocabulary, and reading comprehension. Both the concept scores and the problem solving scores have been included in the mathematics grade equivalent shown in Table V. The concept scores were included because of directions and questions which must be read in this part of the test.

At first glance the relationships shown

may seem low. However, it must be remembered that reading is only one important factor to be considered in problem solving. There are many factors which accompany the complicated task of solving problems.

TABLE V

COMPARISON BETWEEN ARITHMETIC ABILITY AND READING ABILITY

Groups in Arithmetic	Number	Correlations: Math Grade Equivalent and Vocabulary	Correlations: Math Grade Equivalent and Comprehension
Entire Group	186	0.62	0.67
Upper 27 per cent	51	0.07	0.22
Middle 46 per cent	89	0.06	0.25
Lower 27 per cent	50	0.35	0.35

There is a problem of reading in arithmetic which may have some very definite roots in the primary grades. This concerns additional vocabulary which is introduced in primary arithmetic workbooks and other materials related to arithmetic. Teachers spend considerable time in helping childen learn new vocabulary which is presented in basal readers. As a general rule, however, most teachers do not spend a comparable amount of time on new words and terms which occur in arithmetic. The period which is set aside for numbers or arithmetic is usually used on arithmetical skills and concepts. It seems safe to say that additional time which might be spent on learning new vocabulary and word meanings directly connected with arithmetic would be time wisely used in the classroom.

Relationship of Socio-Economic Status to Arithmetic Achievement

There seems to be some misunderstanding with regard to the effect which socio-economic status has upon children's ability to achieve in arithmetic. For example, it is not uncommon for a teacher to say, "Oh, he is smart enough, but he is from a deprived home." The teacher who makes this remark, or a similar remark, has reference to the economic status of the family and is explaining why the child is doing poorly in arithmetic. There is a reasonable explanation for this mis-concept. Let's examine it through the use of actual data. For example in this study a considerable relationship can be shown between socio-economic status and arithmetic ability. However, when I.Q. level is taken into account it can be seen that at various I.Q. levels the percentage of pupils from each socio-economic group classed as high, average or low achievers is about the same. For example, only 44 of 115 pupils in the lower socio-economic group scored above the median on the Iowa Basic Skills Arithmetic Test; whereas a much higher proportion, 73 to 120 pupils in the upper socio-economic group scored above the median. These data, though revealing, do not show the effect of socio-economic status upon a child's ability to achieve in arithmetic.

What effect does socio-economic status have upon a child's ability to achieve in arithmetic? It was found, after dividing the pupils into I.Q. ranges, that only 28 of the 115 pupils in the lower socio-economic group had I.Q.s in the highest range (110 and above) while 65 of the 120 pupils in the higher socio-economic group had I.Q.s in that range. It can be seen from Table VI that I.Q. is associated with socio-economic status; however, socio-economic status seems to have no effect upon children's ability to achieve who have comparable intelligence.

It can be seen that 82 per cent of the pupils in the lower socio-economic group

TABLE VI

COMPARISON OF ARITHMETIC ACHIEVEMENT WITH FREQUENCY OF OCCURRENCE IN I.Q. LEVELS BETWEEN PUPILS IN HIGH AND LOW SOCIO-ECONOMIC GROUPS

I.Q. Range	Number of Pupils: High Economic Status	Number of Pupils: Low Economic Status	Per Cent of Pupils Above Test Median: High Economic Status	Per Cent of Pupils Above Test Median: Low Economic Status
Below 90	6	31	16	3
90–110	44	53	34	36
Above 110	65	28	83	82

who have I.Q.s of 110 or higher scored above the median and 83 per cent of the pupils in the higher socio-economic group scored above the median. Thus, although pupils of higher intelligence are more numerous in the higher socio-economic level, children of like intelligence can be expected to achieve equally in arithmetic regardless of their socio-economic status.

A further study was made to see if any significant differences could be found in out-of-school activities which would contribute to the background of quantitative experiences of either group. No appreciable difference could be found in the out-of-school experiences of either group which would give them added advantage in arithmetic.

EDITOR'S NOTE. Professor Erickson asks us to consider the sub-groups found in our classes. What are the better approaches and learning procedures for various kinds or types of children? Should we expect the pupil of lesser ability to follow the same path as the brighter child but give him more time to reach the goal? We need to face this problem in every classroom. The differences among children which affect learning in arithmetic are not just those of intelligence and related factors. While some of Erickson's correlations may be a bit spurious because of the shape of the distribution of some of the data, the differences in groups which he has noted should cause us to ponder. Someone should use his procedures with a different population to verify the results so that there can be no doubt that his sample was adequate. It was not a small sample. He has given a number of suggestions which should be noted. Certainly, we should spend the necessary time to teach concepts and the necessary vocabulary and the uses and significance of concepts.

BOOK REVIEW

Fun With Mathematics (reprint), Jerome S. Meyer. New York: Fawcett Publications, Inc., 1957. Paper, vii+176 pp., $0.50.

This paperback reprint is chuck full of readable mathematics pitched at the high school level. Most of the material should prove interesting to the better student, and teachers are likely to find it a source for enlivening their presentation of routine subjects. The text is clarified by a number of well chosen diagrams. A wide assortment of topics is included but the arrangement is haphazard. Simplicity of exposition is achieved by restricting the scope of the topics treated. If the reader is interested in a fuller treatment he will have to look elsewhere. Unfortunately, no such bibliography for wider reading is provided. But within the scope of this little volume the discussion is accurate and well written. This reviewer did not spot any misprints, so the teacher should feel no compunction in placing it in the hands of a student.

In addition to a routine exposition of Roman numerals, a variety of interesting puzzles are included. A binary system of numeration is exhibited, and the topic is tied in with the ancient doubling-and-halving method of multiplication of the Egyptians as well as the connection with modern digital computers.

In the discussion of magic squares a number of properties beyond the definition are included, and again some recreational (crossword) puzzles are attached. Computational devices that exploit the base ten of our number system include, besides the familiar casting out nines, curiosa which arise from repeating decimals.

This reviewer found the discussion of Fibonacci numbers particularly interesting. The student is encouraged to discover various recursion formulas for himself. A connection with the Golden Section is disclosed without introducing continued fractions.

A miscellaneous assortment of relations belonging to the Theory of Numbers are included, the selection being such as might awaken interest with no attempt at thoroughness.

The section on trigonometry includes infinite series and may be more difficult than other portions. Nomographs and slide rules are well presented. Several useful formulas for applying mathematics to the geometry of the earth such as sun dial construction are included.

In conclusion, a few problems and fallacies are solved and resolved. The contents seem to justify the title: you can have fun with mathematics.

ARTHUR BERNHART

An Experimental Study in Teaching Percentage

Russell A. Kenney and Jesse D. Stockton*
Kern County Schools, Bakersfield, Calif.

Many students are troubled with percentage. The difficulty may lie in the way in which it is presented. Survey tests in Kern County and elsewhere have indicated a lack of achievement in this particular field. Many teachers have indicated their awareness of the problem in teaching clear concepts in the field of percentage. Supervisors and consultants have observed that many teachers and pupils have difficulties in this field. In order to study the problem at some length and determine if possible, remedial measures, the following study was undertaken.

Since the concept of percentage and its use is first taught in the seventh grade, the selection of personnel involved was from this grade. Administrators and teachers were contacted to find those who might be interested in participating in an experimental study of the methods of developing the concepts of percentage and its use. Many administrators and teachers were contacted personally by the author to determine whether they would be interested in participating in such a study. Some 25 or 30 teachers indicated not only a willingness but a desire to participate. Some were unable to be included in the final experimental group due to other commitments.

A preliminary meeting was held of the foregoing individuals to draw up criteria and to set up the format for the study. At this meeting the following purposes were set up: 1—to improve teaching of percentage, 2—to evaluate the relative values of different methods of teaching percentage. Those present also agreed that in order to achieve these purposes it would be necessary for the group to utilize two or three different methods of teaching percentage and compare the results. Drill or rote teaching without the assistance of any aids whatsoever was one of the approaches proposed. A second procedure suggested was to develop the understandings through the use of all kinds of aids without drill or rote memorization of rules. A third method might be that of a composite of the foregoing.

In order to achieve an evaluation of the relative merits of these three approaches, the teachers agreed to divide into three groups with an attempt to balance the ability of the classes in each group so that they might have comparable results. This was accomplished by comparing the known facts at this time subject to revision when further facts were known.

A four-week experimental period for learning percentage was agreed upon.

The evaluation was to be accomplished through the use of a testing program. Research was inaugurated to find a testing program that might give a pre-test, a closing test and a follow-up test of equal difficulty so that the results might be compared at each of these periods. Committees were ap-

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pointed to take care of the details and report at our next meeting.

At the second meeting of the committee the following general agreement was reached. The experimental period would begin the first week in January, that the arithmetic classes shall meet five times a week, for approximately forty-five minutes a day or as close thereto as possible, and that the test at the end of the experimental period would be on the twenty-first day after the beginning test. It was further agreed that each teacher would be responsible in the testing program for the following items: (a) to see that each pupil takes an achievement test just before the experimental period, immediately following the experimental period, and one the week of April 7 to 11, following the Easter vacation, (b) to pronounce the words or read sections of the tests for those children unable to read the same, but under no circumstances to explain the meanings, (c) to score the tests, (d) to make an item-analysis of the tests, (e) to secure a valid intelligence test given within the last three years, and report the same in terms of I.Q. with the children's ages as of January 6 and to report the results of all tests to R. A. Kenney for each pupil on forms provided by the latter. Each teacher further agreed not to teach percentage after the second test until after the follow-up test in April.

The committee further agreed on the following methods for each of the three groups: GROUP I would emphasize *drill procedures* with reliance on rules and repetition. No understanding or explanation of why was to be given. Books with explanations of reasons for procedures were not to be used.

GROUP II was to emphasize *understandings* and mathematical *reasonings*. Reliance was to be on the building of understandings of why and how. No rules were to be taught as such. No exercises or drill in abstract numbers were to be undertaken. Books, if used, were not to be used for practice.

GROUP III, or control group, was to use a *composite of the two foregoing* methods. Emphases on procedures outlined in the Teacher's Edition of the state text were to be used and the program would include procedures to develop understandings as well as drills to fix and facilitate learning.

Some twenty or twenty-five standardized tests were examined but all were found inadequate for the purposes of this study. Items for special tests were collected and grouped according to concepts and difficulty and placed in a preliminary examination. Seventeen administrators agreed to have these tests administered to their eighth grades in order to obtain the relative difficulty of each problem. The teachers of these schools administered the test to approximately two thousand eighth graders in December of 1957. The test consisted of two parts, one of which was on problem solving and the other was on abstract examples. The problem solving test consisted of 25 groups of three problems each. Each of these problems in a group was of approximate equal difficulty and dealt with the same concept or skill. There were 30 such groups of examples in the other part of the test. Each teacher was furnished with a key to the answers for these test items and an analysis sheet. Each teacher corrected her own papers and recorded the number of correct responses by problems on the item-analysis sheet. These results were turned in to the County Office for analysis. After eliminating those items that seemed to be out of line, the remainder were equated in difficulty as determined from the number of correct responses from the above testing program. From these items three forms of a test in each area were prepared so that they were of equal difficulty.

The seventh graders in the experimental program were equated by using the Horn formula to determine the academic expectancy of each,

$$\left(\text{A.E.} = \frac{2MA + CA}{3}\right)$$

By eliminating the three top ranking students in Group II and one highest and the one lowest pupil in Group III, we were able to establish three groups whose lower and

upper limits were within three months. The median of all three groups fell within a one month interval. The quartile marks were also within a three month range. This arrangement permitted us to have a group of 165 students in Group I, 133 students in Group II, and 177 students in Group III with comparable expectancies.

Form A Test in Problems and Examples, was administered to each of the classes in the first week of January, 1958. The test papers were scored and the number of correct responses for each pupil noted on a form provided for that purpose. The experimental teaching program was then inaugurated and was continued for exactly 19 school days. During this period each of the teachers conscientiously followed the instructions that were developed and accepted at the beginning of the experiment. On the 21st day after the preliminary test, Form B of the same two tests was administered and scored as before above.

During the interval between this test and the follow-up test which was administered during the week following Easter, no instruction was given in percentage to any of these classes. Form C of each of the above tests was given about April 8 for a final check on the development of this experiment. The results of these three tests were then turned in to the County Office for analysis.

The arithmetic mean of the number of correct responses for each of these tests by groups was then computed. The arithmetic mean of each of the quarters of each group was also determined in order to study the effects on the different methods on pupils with different academic expectancies. (Quarters were used in the following manner: first quarter consisted of that group which fell below the first quartile, the second

TABLE I

RESULTS ON EXAMPLES TESTS A AND C

Quarter†	Test A			Test C			Difference in Means	σ_{DM}
	No. of Cases	Arith. Means	S.D.	No. of Cases	Arith. Means	S.D.		
				Group I—Drill				
1	42	3.64	2.30	40	5.70	3.93	2.06*	.72
2	41	4.71	2.45	42	8.17	4.61	3.46*	.81
3	42	5.62	3.41	41	9.98	5.38	4.36*	.99
4	40	8.10	5.01	37	15.51	5.58	7.41*	1.21
Totals	165	5.61	4.01	160	9.25	6.17	3.64*	.56
				Group II—Understanding				
1	33	3.04	2.64	33	4.83	4.67	1.79	.93
2	33	4.70	3.90	33	7.33	5.67	2.63	1.20
3	33	7.70	4.80	33	12.67	6.55	4.93*	1.41
4	33	10.33	6.15	33	16.64	7.28	6.29*	1.66
Totals	132	6.54	5.24	132	10.35	7.64	3.81*	.81
				Group III—Composite				
1	45	3.78	2.84	44	5.84	4.33	2.06*	.78
2	45	5.60	2.79	45	9.67	4.90	4.07*	1.05
3	45	7.67	3.37	45	13.44	5.52	5.77*	.96
4	44	9.66	3.32	45	15.82	4.88	6.16*	.88
Totals	179	6.71	3.39	179	11.22	6.21	4.51*	.53

* Significant at the 1% Level of Confidence.

† Each group was subdivided into four subgroups or quarters by academic expectancy. Quarter #1 refers to that group below the first quartile, Quarter #2 refers to those between first quartile and the median, etc.

quarter included those students who fell between the first quartile and the median, the third quarter were those between the median and the third quartile, and the fourth quarter were those above the third quartile.) The standard deviations for each of these scores were also found in order to determine how significant the gains might be. The differences of the means between various tests and groups were then determined and the statistical significance of each was also computed. The levels of confidence were then checked by accepted formula. The results are shown in the accompanying tables.

In Table I the results for the test in Examples are shown as far as Test A and Test C are concerned. An examination of the table will show that there was a growth, or difference in the means of the two tests, in Group I of 3.64 examples, in Group II of 3.81 examples and in Group III of 4.51 examples. The growth by quarters was also determined and the results are shown in the same table. In Group I they range from 2.06 to 7.41. In Group II they range from 1.79 to 6.29, and in Group II they range from 2.06 to 6.16. In all cases but two these are significant at the 1% level. This is to say that there is less than once chance in 100 that these differences are due to pure chance. It is interesting to note that the difference in the means in the first and second quarters of Group II was not statistically significant.

In Table II the results of the tests in problem solving are shown in a similar fashion with somewhat similar results. We find that the difference in the means of all quarters and the totals are significant except for the first quarter or lowest group. In all three major groups we find the gains were significant during this period. For those above the first quarter, the gains were significant in all sub-groups or quarters. Table III shows the means in the achievement in Examples, comparing Groups I and II. The differences in the means by quarters in Test A ranged from .01 to 2.23 examples, an average for the total groups of .94. These were not significant at the 2% level of confidence. This would support the evidence already presented that these two groups were of equal ability. In Test C the results from the two groups were approximately the same as in Test A, the differences in the arithmetic means ranging in size from .61 for second quarter group to 2.69 for the third quarter group or 1.10 for the total group. None of these differences were statistically significant. Table IV compares Group I with Group III.

Table V shows the comparison of Groups II and III. Again, in Test A we find that the range for the total group was only .17. If we consider the difference of the means of the quarters, we find that the third quarter made only .03 of a problem difference and the second quarter made .90 of a difference in their means. None of these differences are significant statistically. Again, in the final test, or Test C, the difference in the means was .87 while the differences in the quarters ranged from .77 to 2.34. None of these differences were statistically significant, which would tend to indicate that for the time of the experimental period the differences of the methods of instruction between these two groups were not significant.

As indicated above each group made significant progress in problem solving during the experimental period. Each quarter of each group made significant progress during the experimental period with the exception of the first quarter in all three cases. None of these made significant progress in the month of the experimental study.

Table VI indicates the differences in the means for Groups I and II in problem solving. Here again the differences in the means for each of the sub-groups as well as for the total of the group were not significant in Test A. In the final test, or Test C, we find that the difference in the means of the two groups was 1.92. By quarters they range from .26 to 3.86 problems. Of these only two were significant, that for the total group and the one for the third quarter, both of which were significant at the 1% level.

Table VII indicates the differences in problem solving between Groups I and III. Again the difference in the means of

TABLE II

RESULTS ON PROBLEMS TESTS A AND C

Quarter	Test A: No. of Cases	Test A: Arith. Means	Test A: S.D.	Test C: No. of Cases	Test C: Arith. Means	Test C: S.D.	Difference in Means	σ_{DM}
				Group I—Drill				
1	42	1.95	1.37	40	2.53	1.89	.58	.37
2	41	2.76	1.56	40	4.30	3.34	1.54*	.58
3	42	3.74	2.41	41	6.05	4.23	2.31*	.76
4	40	5.80	2.40	37	10.58	5.34	4.79*	.96
Totals	165	3.48	2.45	158	5.78	4.87	2.30*	.43
				Group II—Understanding				
1	33	2.15	1.13	33	2.79	2.08	.64	.81
2	33	2.94	2.17	31	5.74	5.15	2.80*	1.00
3	33	4.33	2.59	33	9.91	5.94	5.58*	1.23
4	33	7.27	4.59	33	12.24	6.09	4.97*	2.33
Totals	132	4.18	3.52	130	7.70	6.30	3.52*	.63
				Group III—Composite				
1	45	2.27	1.32	44	2.73	1.76	.46	.33
2	44	3.11	1.77	45	4.91	2.43	1.80*	.44
3	44	3.89	2.42	44	7.07	3.34	3.18*	.62
4	44	4.34	2.56	45	8.42	4.05	4.08*	.72
Totals	177	3.37	2.25	178	5.76	3.73	2.39*	.33

* Significant at the 1% Level of Confidence.

TABLE III

EXAMPLES—GROUPS I AND II COMPARED BY TESTS A AND C

Quarter	Group I: No. of Cases	Group I: Arith. Means	Group I: S.D.	Group II: No. of Cases	Group II: Arith. Means	Group II: S.D.	Difference in Means	σ_{DM}
				Test A				
1	42	3.64	2.30	33	3.04	2.64	.60	.58
2	41	4.71	2.45	33	4.70	3.90	.01	.78
3	42	5.62	3.41	33	7.70	4.80	2.08	.99
4	40	8.10	5.01	33	10.33	6.15	2.23	1.38
Totals	165	5.61	4.01	132	6.55	5.24	.94	.55
				Test C				
1	40	5.70	3.93	33	4.83	4.67	.87	1.02
2	42	8.17	4.61	32	7.56	5.67	.61	1.23
3	41	9.98	5.38	33	12.67	6.55	2.69	1.42
4	37	15.51	5.58	33	16.64	7.28	1.13	1.56
Totals	160	9.25	6.17	131	10.35	7.64	1.10	.82

TABLE IV

EXAMPLES—GROUPS I AND III COMPARED BY TESTS A AND C

Quarter	Group I			Group III			Difference in Means	σ_{DM}
	No. of Cases	Arith. Means	S.D.	No. of Cases	Arith. Means	S.D.		
				Test A				
1	42	3.64	2.30	45	3.78	2.84	.14	.55
2	41	4.71	2.45	45	5.60	2.79	.89	.57
3	42	5.62	3.41	45	7.67	3.37	2.05*	.73
4	40	8.62	5.01	44	9.66	3.32	1.04	.94
Totals	165	5.61	4.01	179	6.71	3.78	1.10*	.42
				Test C				
1	40	5.70	3.93	44	5.84	4.33	.14	.90
2	42	8.17	4.61	45	9.67	4.90	1.50	1.02
3	41	9.98	5.38	45	13.44	5.52	3.46*	1.18
4	37	15.51	5.58	45	15.82	4.88	.31	1.17
Totals	160	9.25	6.17	179	11.22	6.21	1.97*	.67

* Significant at the 1% Level of Confidence.

TABLE V

EXAMPLES—GROUPS II AND III COMPARED BY TESTS A AND C

Quarter	Group II			Group III			Difference in Means	σ_{DM}
	No. of Cases	Arith. Means	S.D.	No. of Cases	Arith. Means	S.D.		
				Test A				
1	33	3.04	2.64	45	3.78	2.84	.47	.63
2	33	4.70	3.90	45	5.60	2.79	.90	.80
3	33	7.70	4.80	45	7.67	3.37	.03	.98
4	33	10.33	6.15	44	9.66	3.32	.67	1.18
Totals	132	6.54	5.24	179	6.71	3.39	.17	.52
				Test C				
1	33	4.83	4.67	44	5.84	4.33	1.01	1.04
2	32	7.33	5.67	45	9.67	4.90	2.34	1.24
3	33	12.67	6.55	45	13.44	5.52	.77	1.41
4	33	16.64	7.28	45	15.82	4.88	.82	1.56
Totals	131	10.35	7.64	179	11.22	6.21	.87	.81

these two groups in this test were not significant except for the fourth quarter which again would indicate the parity of these two groups with the possible exception of the fourth quarter.

In Test C, or the final test for these two groups, again, the difference of the means between these two was not significant at the 2% level, however, the fourth quarter did achieve a difference in the means in favor of Group I which was significant at the 5% level. In Test C we find much the same difference.

Group II, Table VIII, achieved a means

TABLE VI

PROBLEMS—GROUPS I AND II COMPARED BY TESTS A AND C

Quarter	Group I No. of Cases	Group I Arith. Means	Group I S.D.	Group II No. of Cases	Group II Arith. Means	Group II S.D.	Difference in Means	σ_{DM}
				Test A				
1	42	1.95	1.37	33	2.15	1.31	.20	.75
2	41	2.76	1.56	33	2.94	2.17	.18	.45
3	42	3.74	2.41	33	4.33	2.59	.59	.58
4	40	5.80	2.40	33	7.27	4.59	1.47	.88
Totals	165	3.48	2.45	132	4.18	3.52	.30	.36
				Test C				
1	40	2.53	1.89	33	2.79	2.08	.26	.47
2	40	4.30	3.34	31	5.74	5.15	1.44	1.07
3	41	6.50	4.23	33	9.91	5.94	3.86*	1.23
4	37	10.59	5.34	33	12.24	6.09	1.56	1.38
Totals	158	5.78	4.87	130	7.70	6.30	1.92*	.67

* Significant at the 1% Level of Confidence.

TABLE VII

PROBLEMS—GROUPS I AND III COMPARED BY TESTS A AND C

Quarter	Group I No. of Cases	Group I Arith. Means	Group I S.D.	Group III No. of Cases	Group III Arith. Means	Group III S.D.	Difference in Means	σ_{DM}
				Test A				
1	42	1.95	1.37	45	2.27	1.32	.32	.29
2	41	2.76	1.56	44	3.11	1.77	.35	.36
3	43	3.74	2.41	44	3.89	2.42	.15	.52
4	40	5.80	2.40	44	4.34	2.56	−1.46*	.54
Totals	165	3.54	2.45	177	3.37	2.25	− .17	.25
				Test C				
1	40	2.53	1.89	44	2.73	1.76	.20	.40
2	40	4.30	3.34	35	4.91	2.43	.61	.64
3	41	6.50	4.23	44	7.07	3.34	1.02	.83
4	37	10.86	5.34	45	8.42	4.08	−2.44	1.07
Totals	158	5.78	4.87	178	5.76	3.73	− .02	.48

* Significant at the 1% Level of Confidence.

considerably above that of Group III and statistically significant for the third and fourth quarters of the total group.

Table IX shows the net gain by groups in the mean number of correct responses from Test A to Test C. The table should be interpreted in this way. In Examples, Group I made a mean gain of 3.64 correct responses between Test A and C. A cursory examination of this table will indicate as shown above that in every case, significant gains were made except the lower half of Group

II in Examples and the lower quarter in all three groups in problem solving.

We might evaluate the effectiveness of each of these methods by comparing the net gains made by each group over the same two tests. This is shown in Table X. If we compare Groups I and II the difference for the total group in the net gain was only .17 of one response. This would be statistically significant at the 5% level. If we examine the net gains by quarters of each group, we find that the second quarter as defined above and the fourth quarter made significant gains at the 1% level while the third quarter made a significant gain at the 2% level. The negative signs indicate that the second group named made the least gain. In other words, in comparing Group I with Group II, Group I as a whole made more gain than Group II. In the second pair of columns we have compared Group I with Group III. The lower quarter of each group made exactly the same progress. Second quarter and the third quarter of Group III did significantly better than Group I, while the fourth quarter Group I made a significantly better gain than Group III. In the last two columns we have compared Group II

TABLE VIII

PROBLEMS—GROUPS II AND III COMPARED BY TESTS A AND C

Quarter	Group II			Group III			Difference in Means	σ_{DM}
	No. of Cases	Arith. Means	S. D.	No. of Cases	Arith. Means	S. D.		
				Test A				
1	33	2.15	1.31	45	2.27	1.32	.12	.75
2	33	2.94	2.17	44	3.11	1.77	.17	.46
3	33	4.33	2.59	44	3.89	2.42	−1.44*	.58
4	33	7.27	4.59	44	4.34	2.56	−2.93*	.89
Totals	132	4.18	3.52	177	3.37	2.25	− .81	.35
				Test C				
1	33	2.79	2.08	44	2.73	1.76	− .06	.45
2	31	5.74	5.15	45	4.91	2.43	− .83	.99
3	33	9.91	5.94	44	7.07	3.34	−2.84*	1.15
4	33	12.24	6.09	45	8.42	4.05	−3.82*	1.22
Totals	130	7.70	6.30	178	5.76	3.73	−1.94*	.62

* Significant at the 1% Level of Confidence.

TABLE IX

DIFFERENCES (GAINS) IN MEANS OF CORRECT RESPONSES BY GROUPS FROM TEST A TO TEST C

Quarter	Examples			Problems		
	I	II	III	I	II	III
1	2.06*	1.79	2.06*	.58	.64	.46
2	3.46*	2.63	4.07*	1.54*	2.80*	1.80*
3	4.36*	4.93*	5.77*	2.31*	5.58*	3.18*
4	7.41*	6.29*	6.16*	4.79*	4.97*	4.08*
Totals	3.64*	3.81*	4.51*	2.30*	3.52*	2.39*

* Significant at the 1% Level of Confidence.

TABLE X

DIFFERENCES (GAINS) IN MEANS BETWEEN GROUPS

Quarter	I–II		I–III		II–III	
	Difference	DM	Difference	DM	Difference	DM
			Examples			
1	−.27	.1980	0.0	.1622	.27	.1997
2	−.83*	.2429	.61*	.1995	1.44*	.2604
3	.57	.2897	1.41*	.2100	.84*	.2851
4	−1.12*	.3509	−1.25*	.2366	−.13	.3173
Totals	.17	.0825	.85	.3975	.70	.4012
			Problems			
1	.06	1.0830	−.12	.8344	−.18	1.0664
2	1.26	1.2575	.26	1.0132	−1.00	1.2019
3	3.27*	.9412	.87	.6164	−2.42	1.3227
4	.18	1.5113	−.71	1.2934	−.89	1.4297
Totals	1.22	1.0314	.09	.8710	−1.13	.9791

* Significant at the 1% Level of Confidence.

and Group III. The second and third quarters of Group III did significantly better than Group II. In summarizing this material we can therefore say that in comparing the middle 50% of the class, Group III did better than Group I or Group II. In the lower part of this table, the relationships are shown as far as problem solving is concerned. The differences in net gains in problem solving were not significant in any case except for the third quarter of Group II which made a significantly better showing on the second test than did Group I.

Summary

In summarizing the above results we may tentatively conclude that: (1) The three upper quarters of all three groups made significant progress during the experimental period. The progress is measured in the difference of the means of the achievement at two testing periods and indicates a significant difference at all levels except the first quarter sub-group of each major group. (2) The differences in the means between the different groups as measured by the Form A Tests was not significant in most cases. (3) There is some evidence to indicate that the composite method of Group III was more effective than the drill method of Group I with the total group and especially with the third quarter sub-group. (4) The emphasis given to understanding in Group II seems to have had some advantage in problem solving for the third and fourth quarter groups as evidenced in Table VIII. (5) The lack of statistically significant results in some parts of this experiment may be due to the fact that the experimental period was too short. (6) Another possibility might be that there is no significant difference in the teaching methods used. (7) The evidence leads to the conclusion that perhaps the experiment should be carried on for a longer period, either the period of instruction lengthened or the same amount of instruction spread over a longer period of time.

Addenda

In order that others may study the results of the experiment and perhaps compare results, the question used on the Problems Test—Form C are given below. Also, the scores by groups on this test are given.

PERCENTAGE TEST—PROBLEMS, FORM C

1. If a number is divided by another number less than one, is the quotient larger or smaller than the dividend?

2. What is the value of $\frac{7}{8}$ to the nearest hundredth?
3. How many times as much as .05: is 5?
   is 2.5?
   is .35?
4. How many boards 3.5 feet long can be cut from a board 14 feet long?
5. What is 100% of 37?
6. If apples contain 85% water, what per cent of them is solid matter?
7. Which is most, $\frac{3}{8}$, 40%, or .35%?
8. Which is the least, 3%, .3, or 2.5%?
9. Express 80% as a fraction in lowest terms.
10. 14.3% is equal to the decimal—14.3, 1430, .0143, .143, 1.43
11. A department store ran a sale, marking the price of every article down 30%. What was the sale price of an article originally marked $15? $14.70, $12, $10.50, $10, or $5
12. A bank pays $\frac{1}{2}$% interest on its deposits at each interest period. If the deposits are $365,000, what is the amount of the interest?
13. A family spends 30% of its $2000 annual income for food. What is the average monthly cost of food?
14. If a man earns $80 in a week and has deductions of 1% for unemployment insurance, $1\frac{1}{2}$% for old-age security, and $12 for income tax, how much does he have left? $65.50, $66, $67.80, $67.97, or not given
15. A man received seven per cent interest on a loan of $200.00 for 6 months. How much interest did he receive? $10.00, $7.00, $3.50, $4.50, or none of these
16. Find the interest on $360 at 2% for 6 months.
17. Goods that cost $60 were sold for $96. If the combined overheads were 25% of the selling price, what was the profit?
18. A dress selling for $26.00 was marked to give a margin of 20% of the selling price. What was the cost?
19. If a real estate agent's commission is 5% of the selling price, what does he receive for selling a house for $15,000? $75.00, $300.00, $333.00, $750.00, or not given
20. Mrs. Tucker bought a window fan priced at $54.50. She received a discount of 20%. How much did the fan cost her?
21. A note for $500 was discounted at 6% for 30 days. What were the proceeds?
22. Frank earned $16.00 and saved $8.00 of it. What per cent did he save? $\frac{1}{2}$, 50, $33\frac{1}{3}$, $24.00, or none
23. When the price of a dozen eggs increases from 25¢ to 35¢, what is the per cent of increase?
24. BJHS won 26 basketball games while playing 40 games. What per cent of the games played did they win?
25. A shirt marked to sell at $4.00 was sold for $2.80. What was the per cent of reduction?

FREQUENCY CHART—PROBLEMS TEST, FORM C

Scores	Group I	Group II	Group III
25			
24		1	
23		1	
22	1	1	
21	1	2	
20	1	5	
19	2	2	
18	0	2	1
17	2	3	2
16	1	6	1
15	4	2	5
14	5	4	2
13	6	2	3
12	1	2	2
11	1	3	3
10	5	7	4
9	5	4	6
8	3	4	14
7	9	5	16
6	15	5	20
5	15	8	29
4	13	18	17
3	24	15	22
2	24	16	12
1	12	8	17
0	8	4	2
Totals	158	130	178

EDITOR'S NOTE. Apparently the better method to use in teaching percentage is a combination of the development of understanding plus sufficient drill or practice to establish the learning at a useful level. It is interesting and a little shocking to note what small gains were made by the lower levels during the 19 days of instruction. Note in the Addenda the frequencies of scores made by the three groups on the Problems Test—Form C. Note where the high frequencies of each group occur. Would your school do better after 19 days of instruction followed by six weeks of no instruction or practice? The editor is more concerned with gross score gains from one testing period to another than he is with the proposition of ruling out "chance" as a statistical factor. He would have expected higher gains from Test A to Test C for each of the three groups. Some of the problems in the test normally would not be covered in a first four-week teaching period of percentage. But the test was not too difficult for a few pupils. If we want pupils to learn any topic in arithmetic we must provide the stimulation, the teaching, the learning, and the final checks. The Kern County teachers and their leaders have given us a study of comparison of three approaches from differing points of view. We can build upon their results.

The Role of Attitude in Learning Arithmetic

J. PETER FEDON
The Tatnall School, Wilmington, Delaware

THIS IS A PARTIAL ANALYSIS of an arithmetic teaching experiment conducted in the third grade of an independent Wilmington School during the school year 1957–58.

The group was divided into two sections of 16 children. No effort was made to equate the groups on a one to one basis but an attempt was made to have a heterogeneous selection of about equal ability on the basis of intelligence, reading ability, arithmetic age, chronological age, and sex.

Prior to the experiment each child had been given the Wechsler's Intelligence Scale for Children (WISC). The WISC was administered by a competent psychologist, but while some tests were given in the Fall of 1957 others had been given as far back as kindergarten. The arithmetic age was determined by the California Arithmetic Achievement Test. Reading ability was based on observed scholastic achievement as indicated by the second grade teachers. Each group consisted of 9 boys and 7 girls. The chronological ages of the two groups ranged from 7 years, 6 months to 9 years 2 months with a mean of 8 years 6 months. The intelligence quotients ranged from 99 to 140 with a mean of 115.6. The arithmetic ages ranged from 8 years, 1 month, to 9 years, 11 months with a mean of 9 years, 0 months.

The purpose of this paper is to report on only one aspect of this larger study. An attitude inventory was developed in an effort to "measure" attitudes toward arithmetic.

In reporting on the results of the attitudes, the findings will be discussed as a whole rather than as a comparison of one group as opposed to the other. The important factors are the attitudes of children and the method used in measuring these attitudes.

The study was inspired by two similar studies conducted by Wilbur H. Dutton (1 and 2). He attempted to show reactions of one group of Junior High School students and another group of College Students. It was found that initial strong feelings both for and against arithmetic were formed in or around the third grade.

Dutton's work was based on the original work of Thurstone (5) who outlined procedures for building a scale that would "measure" attitudes.

The attitudes were measured by accepted or rejected opinions. Thurstone (3, p. 535) refers to this technique as the "more or less" type of judgment. He defends the attempt to measure attitudes on the basis that man's total feelings and inclinations are indicated through verbal expressions if the situation is free of pressure or inhibitions. Thurstone (3, pp. 531–37) claims that "an opinion symbolizes an attitude" and "serves as the carrier of symbol of the attitudes of people."

Assuming then that an atmosphere conducive to honest opinions is established, the attitude scale as prepared by Dutton (2, p. 27) should measure attitudes more objectively since it is constructed on a rational base line so designed as to interpolate the statements at a given scale value.

A detailed explanation of how this scale was constructed, scored, and tested for reliability can be found in the two sources mentioned above.

Procedure

It was felt that an "intensity scheme" would increase the accuracy of the reactions. Intensity of feelings was determined by a color chart. In building the "color intensity scheme," seven colors were placed in a circle as shown in Figure I. The circle of colors was used so as not to influence the child's preference through the mechanics of selection.

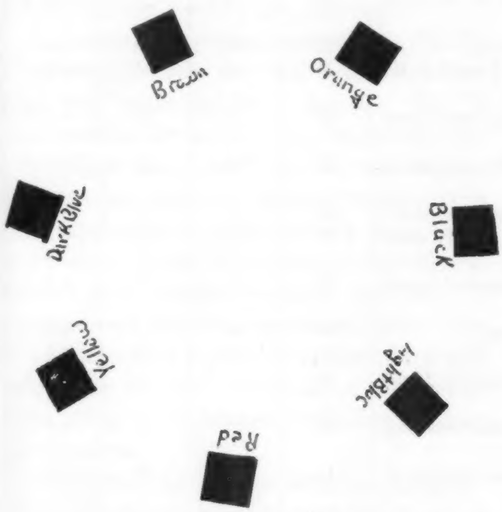

FIG. I. Color selection wheel.

The children were asked to select the most liked, disliked, and neutral colors first and then their intermediate choices.

The choices were scored on a point basis. Allowing (7) points for the most liked, (6) for the next and so on until the seventh choice, or least liked was given (1) point. The selections were then tabulated. Figure II shows the distribution of choices, the number of times each color was selected, and the total point score.

One change was made when making the final "color scheme." Black was placed last and orange was placed in the sixth position, even though the total point score for black was higher. This was justified on the basis of the greater number of times black was chosen last (12) as compared to any other color.

It was now necessary to teach the significance of the "color intensity scheme." It was important to stress the relationship of red to indicate an extreme positive attitude and black to indicate an extreme negative attitude and that yellow conveyed a neutral attitude. The intermediate colors, dark blue, light blue, brown, and orange conveyed lesser degrees of the two extremes.

FIG. II. Distribution of color selections.

NUMBER OF CHOICES

Rank	1	2	3	4	5	6	7	Total Score
Red	11	8	4	2	6	0	1	172
D. Bule	8	8	4	4	2	5	1	157
L. Blue	5	7	6	4	3	4	3	143
Yellow	4	1	7	4	9	3	4	122
Brown	0	4	4	10	2	6	6	108
Orange	1	0	5	4	8	9	5	95
Black	3	4	2	4	2	5	12	99

The children's understanding of the color scheme was tested through common experiences in many situations during the entire school year.

Results of the Responses on the Attitude Scale

Dutton's attitude scale is shown in Figure III. The scale value and the number of each statement, as they were read, are indicated in the left hand columns. The number of responses and the color scale values are indicated in the right hand columns. Each question was read to the children to avoid reading difficulties. In addition, the following words were defined to ensure thorough understanding: No. 3, accurately; No. 5, practical; No. 7, enthusiastic; No. 10, challenge; No. 13, detest; No. 15, avoid; No. 22, value. The children reacted to every attitude. The acceptance or rejection of the attitude was determined by their color choice.

A strong respect for arithmetic was indicated by the children's responses to statements 5, 8, 10, and 14. Statement 8, "Arithmetic is as important as any other subject" and statement 14, "I enjoy doing problems when I know how to work them" are particularly significant for the intensity of the children's reaction as indicated by the red and dark blue responses. There were 18

FIG. III. Responses of third grade children on an arithmetic attitude scale.

Scale Value	Statement Number	Attitude Statement	Number of Responses			Color Scale Value
			Positive	Negative	Neutral	
1.0	13	I detest arithmetic and avoid using it at all times.	10	14	8	−9.5
1.5	20	I have never liked arithmetic.	11	17	4	−8.5
2.0	18	I am afraid of doing word problems.	16	12	4	−7.5
2.5	11	I have always been afraid of arithmetic.	12	17	3	−6.5
3.0	22	I can't see much value in arithmetic.	17	13	2	−5.5
3.2	15	I avoid arithmetic because I am not very good with figures.	13	14	5	−5.1
3.3	9	Arithmetic is something you have to do even though it is not enjoyable.	22	8	2	−4.9
3.7	2	I don't feel sure of myself in arithmetic.	12	10	10	−4.1
4.6	6	I don't think arithmetic is fun, but I want to do well in it.	21	5	6	−2.3
5.3	7	I am not enthusiastic about arithmetic, but I have no real dislike for it either.	15	6	11	− .9
5.6	4	I like arithmetic, but I like other subjects just as well.	17	3	12	− .3
5.9	8	Arithmetic is as important as any other subject.	25	3	4	+ .3
6.7	14	I enjoy doing problems when I know how to work them well.	27	3	2	+1.9
7.0	10	Sometimes I enjoy the challenge presented by an arithmetic problem.	18	6	8	+2.5
7.7	5	I like arithmetic because it is practical.	19	10	3	+3.9
8.1	19	Arithmetic is very interesting.	20	8	4	+4.7
8.6	3	I enjoy seeing how rapidly and accurately I can work arithmetic problems.	23	5	4	+5.7
9.0	12	I would like to spend more time in school working arithmetic.	11	15	6	+6.5
9.5	1	I think about arithmetic problems outside of school and like to work them out.	11	11	10	+7.5
9.8	17	I never get tired of working with numbers.	13	9	10	+8.1
10.4	21	I think arithmetic is the most enjoyable subject I have taken.	16	10	6	+9.3
10.5	16	Arithmetic thrills me, and I like it better than any other subject.	16	12	4	+9.5

red and 6 dark blue choices for statement 8, and 19 red and 6 dark blue choices for statement 14. It was interesting to note that both statements received the greatest number of positive responses and the least number of negative responses.

A strong positive and particularly a strong red response (18 red) in statement 6, "I don't think arithmetic is fun, but I have no real dislike for it either" would seem to indicate that children consider arithmetic as being necessary. The responses to statement 9, "Arithmetic is something you have to do even though it is not enjoyable" with 22 positive responses seems to substantiate this feeling.

The challenge presented by arithmetic is apparently a strong motivating force. Statement 3, "I enjoy seeing how rapidly and accurately I can work arithmetic problems" was positively indicated 23 times. Statement 19, "Arithmetic is very interesting" was positively chosen by 20 children. In both statements most childen indicated strong feelings by choosing the color red.

It was revealing to note the strong negative responses (17 each) to statement 11, "I have always been afraid of arithmetic" and statement 20, "I have never liked arithmetic." The children that rejected these two statements were particularly emphatic. Each received 15 black choices.

Statement 18, "I am afraid of doing word problems" shows some anxiety about word

problems. There were 12 that indicated an intense feeling toward this statement, while 4 chose the dark blue color. It was interesting to note, however, that there were 12 negative responses to statement 18, 10 of which were the color black. Statement 13, "I detest arithmetic and avoid using it at all times" shows that 10 children indicated a disrespect for arithmetic in varying degrees of intensity. By the same token, an equal number of childen chose the color black when reacting toward this statement.

Two extremely positive statements, No. 16, "Arithmetic thrills me and I like it better than any other subject" and No. 21, "I think arithmetic is the most enjoyable subject I have ever taken" shows that 16 children reacted positively to each statement in varying degrees of intensity. To these same statements, 10 and 12 children, respectively, reacted in a negative manner. For No. 13, an extremely negative statement, there were 10 children that reacted favorably. It would seem that at least one-third of the group have already established attitudes opposed to arithmetic.

Other statements that show an unfavorable attitude toward arithmetic are Number 11, "I have always been afraid of arithmetic" with 12 positive responses, 8 of which were red; and Number 15, "I avoid arithmetic because I am not very good with numbers," positively indicated 13 times, 9 of which were red.

Developing a Scale Value for the Color Scheme

In order to combine the values of the color intensity scale and Dutton's statements, it was necessary to evolve a statistically valid method. Dutton's scale went from 1.0 to 10.5 (see Figure III). By taking the median of 5.75 as 0 and then going up and down the scale by .5, we evolve a numerical scale that ranges from -9.5 to $+9.5$ with eleven positive and eleven negative statements on each side. If we arbitrarily assign a value of .7 (red), .6 (dark blue), .5 (light blue), .4 (yellow), .3 (brown), .2 (orange), .1 (black), we then get an intensity value for each statement. When we multiply the rating of each statement (-9.5 to $+9.5$) by the color scale values (.1 to .7) we can get a measure of intensity for the entire attitude scale. The total positive and negative scores are then subtracted and the resultant remainder is the child's attitude for Dutton's entire scale.

The possible total score ranges from $+36.42$ to -32.58. These are broken apart to correspond to the color spread. Figure IV shows this spread, while Figure V summarizes the number of cases for each attitude classification.

Related Research

Along with the attitude scale, the children were asked to react to some general questions. They were asked to indicate what they felt was their teacher's, their parents', and their own attitudes toward arithmetic. This was an attempt to weigh additional factors

Fig. IV. Color spread for attitude category.

Number Spread	Color Category
+36.42 *to* +26.02	Red
+26.01 *to* +15.61	Light Blue
+15.60 *to* + 5.20	Dark Blue
+ 5.9 *to* − 4.65	Yellow
− 4.66 *to* −13.96	Brown
−13.97 *to* −23.27	Orange
−23.28 *to* −32.58	Black

FIG. V. Number of cases summarized

General Feeling	Color	No. of Cases
Strongly Positive	Red	2
Positive	Dark Blue	5
Slightly Positive	Light Blue	6
Neutral	Yellow	11
Slightly Negative	Brown	7
Negative	Orange	1
Strongly Negative	Black	0
Total Cases		32

that have a direct bearing upon the attitudes.

The children's attitudes were solicited after working on tests of problem solving and arithmetic processes. These tests were standardized diagnostic forms that culminated each chapter in the basic textbook. After completing each test, the children indicated on the "color scheme" their feelings toward arithmetic. Four such tests were administered during the year beginning in January. A standardized achievement test was administered in March instead of the usual test. The results for these tests are listed in Figures VI and VII.

"Attitudes toward problem solving," Figure VI, seems to indicate a definite shift. Whereas a strong yellow feeling was characteristic during the January, February and March tests, there is a pronounced positive swing in April and May. It is particularly significant for the large increase in the red responses during the latter two tests.

As shown in Figure VII, there seemed to be little change in children's attitudes toward the "arithmetic processes" test. The March standardized test showed a large number of black responses which can probably be attributed to the graduated difficulty of the problems in the test itself. The April and May tests show a wide distribution on the "color scale" with a consistent display of feelings both for and against arithmetic.

The children's attitudes were grouped into five categories as shown in Figure VIII. The results of this classification are listed in Figure IX.

In Group I, or those who were classified as strongly positive, the color red was marked most frequently. Whenever a child would change to an intermediate color, the shift was usually to dark blue.

Of the eleven who indicated a strong positive attitude on the processes tests, Figure IX, eight achieved good scores constantly. The scores of three children varied from good to fair. Of the twelve who indicated a strong positive attitude on the problem tests all achieved a high degree of success.

Group II made greater use of the intermediate colors as varying degrees of difficulty were encountered. Success on any given test more often determined the attitude. However, the neutral color was fre-

FIG. VI. Attitudes toward problem solving tests.

	Attitude	Red	Dark Blue	Light Blue	Yellow	Brown	Orange	Black
Monthly Tests	January	12	2	6	10	0	0	2
	February	9	4	3	12	2	1	1
	March Standardized Achievement Test	8	3	2	14	0	3	2
	April	17	3	2	7	0	1	2
	May	20	4	2	4	1	1	0
	Totals	66	16	15	47	3	6	7

FIG. VII. Attitudes toward arithmetic processes tests.

	Attitude	Red	Dark Blue	Light Blue	Yellow	Brown	Orange	Black
Monthly Tests	January	11	4	4	10	2	0	1
	February	16	4	4	6	1	1	0
	March Standardized Achievement Test	10	4	1	8	0	0	9
	April	17	5	1	4	2	1	2
	May*	12	4	4	6	1	2	3
	Totals	66	21	14	34	6	4	15

quently employed regardless of the scores achieved. The general achievement of the children in this group was good.

Within Group III, three classifications of equal grouping could be made. One group was consistently neutral while the other two groups showed slight leanings in either direction. In all cases, however, it was obviously clear that there was an indifferent attitude toward arithmetic. The scores achieved were good but had little effect upon influencing the attitude chosen.

In Group IV, the attitudes were inconsistent. There was no relationship between the color selected and success achieved, nor any revealing pattern from one month to the next. It was not uncommon to switch from one extreme to another. Group V was definite in its dislike of arithmetic. The only fluctuation in their negative attitude was among the three colors that indicated a dislike for arithmetic. It was interesting to note that of the four in this group, three were very high achievers in arithmetic.

A final observation on the results of Figure IX is that the four negative attitudes were indicated on the "processes" tests while none could be classified in this category on the arithmetic "problems" tests.

FIG. VIII. Grouping on the basis of attitudes toward achievement tests.

Group	Classification
ONE	Those who were consistently positive and indicated a strong attitude toward arithmetic.
TWO	Those who were positive but occasionally had neutral reservations. Their general attitude was not particularly strong.
THREE	Those who indicated a neutral attitude and showed no obvious tendency in either direction.
FOUR	Those who were inconsistent in their responses and no clear pattern was discernible.
FIVE	Those who were obviously negative in their attitudes toward arithmetic.

FIG. IX. Classification based on tests of processes and problems.

Group	Number of Cases		Total
	Process Tests	Problem Tests	
Group I (Stongly Positive)	11	12	23
Group II (Positive)	9	7	16
Group III (Neutral)	3	7	10
Group IV (Inconsistent)	5	6	11
Group V (Negative)	4	0	4

FIG. X. Children's attitudes of how parents and teachers feel toward arithmetic.

Attitude	Red	Dark Blue	Light Blue	Yellow	Brown	Orange	Black
Father's Attitude	11	1	5	11	2	1	1
Mother's Attitude	10	5	5	8	2	0	2
Teacher's Attitude	22	4	1	4	0	0	1
Total	43	10	11	23	4	1	4

Figure X shows children's responses when asked to indicate how their parents and teachers felt toward arithmetic. The children felt that parents' attitudes were generally favorable although a large number were undecided as was indicated by their neutral responses. However, there can be no doubt about how the children think their teachers feel toward arithmetic. Particularly significant here is the large number of red responses (22) and only one negative response.

Summary

It is hoped that by studying children's attitudes we may be able to evaluate the total arithmetic program more intensively.

The scales discussed in this paper are an attempt to cast light on some aspects of the arithmetic curriculum that are liked and disliked. Attitudes play an important part in the success of the arithmetic program. If we feel that they are a valid criteria for evaluating the effectiveness of our program, then the application of this scale will provide better opportunities to study children's reactions as they experience arithmetic in daily life.

The initial results obtained through this attempt to measure attitudes have given enough encouragement to extend this study. Some general observations that were made are:

1. It would seem apparent that very definite attitudes are being expressed, both for and against arithmetic as early as third grade.
2. Dutton's attitude scale was adequate. However, it would be more practical to build an attitude scale with elementary school children participating in its structure.
3. The "color scheme" provided an opportunity for greater discrimination in accepting or rejecting an attitude.
4. There is a general feeling that various aspects of arithmetic are enjoyable and necessary, but not always meaningfully significant.

Bibliography

1. Dutton, Wilber H., "Attitudes of Junior High School Pupils Toward Arithmetic," *The School Review*, (January, 1956) pp. 18–22.
2. Dutton, Wilber, H., "Measuring Attitudes Toward Arithmetic," *The Elementary School Journal* (September, 1954) pp. 23–31.
3. Thurstone, L. L., "Attitudes Can be Measured," *The American Journal of Sociology*, Volume XXXII, No. 4 (January, 1928) pp. 529–54.

EDITOR'S NOTE. Yes, attitude plays an important role in learning arithmetic. Mr. Fedon points out that, at the level of grade three, children have already rather firm attitudes toward the subject. It was good to learn that these pupils thought that their teachers had a very favorable attitude. How are attitudes formed? Just how important a factor is the casual comment of parents? How do we as teachers work for establishing a wholesome frame of mind with our pupils? Is it necessary to do this consciously? How can a school overcome the unfavorable impressions gained from a perhaps thoughtless parent who says, "I hope you can learn arithmetic, I never could," or "I don't understand why you have to learn all that stuff"? Occasionally a conflict comes in the other direction, as for example, the pupil who learned at school that $2 \div 0 = 2$ and whose father tried to explain the significance of division by zero only to have the child say to his mother, "I agree with father at home to make him feel good but at school we know that two divided by zero is two." Perhaps other teachers have different methods of determining the feelings of their pupils toward arithmetic which they would like to report. How does one teacher take a whole grade that last year "hated arithmetic" and in a few short months create a favorable change?

Does she?

Utilizing the Strategic Moment In Arithmetic

Mary Ballard Pieters
Claremont Elementary School, Claremont, Calif.

The Search

Billy was four years old. The sand dropped more and more slowly from his dipper as he looked up at the blue sky with the clouds moving quietly over. His voice became softer as he spoke.

"Houses don't run into stars—
They're higher than that.
Trees don't run into stars—
They're higher than that.
Airplanes don't run into stars—
They're higher than that.
And maybe
They're higher than that!"

What sort of quest is this that makes Billy reach out into the world of the known to its very outer limits, and then to search beyond? And in what way will he be helped in this adventure by the teacher lucky enough to hear and know his wonder?

Few upper-grade teachers have the opportunity of the nursery-school and kindergarten teacher to catch these glimpses of the reaching mind. Even primary teachers voice a wish that children could be as natural with them as with the kindergarten teacher who may see what's deeply, really there. Somehow, somewhere in the early school years, children learn to fit into some outer scheme the social situation imposes upon them, a scheme that precludes or, at best, limits the openness of experimenting into the world of wonder that is their own.

Yet this world is not far beneath the surface. High school students, trained in observing in kindergartens, are often deeply moved by what seems to them an unbelievable discovery—that these "babies" are trying just as eagerly, just as violently, as they themselves to break through to something that is there to be discovered.

Is it only the social situation and its pressures that make this wonder lose itself, go underground soon after school entrance, not to show itself in a class situation, even perhaps as far as high school? Or is there a lack of sequential development in curriculum, a lack of recognition of what the child is seeking, that separates the child's search and what the curriculum offers him to aid in his quest?

The Curriculum

It has been held that during the early school years a rich experience program would permit this search to continue. From the viewpoint of the teacher who wishes to utilize the strategic moment for teaching to the fullest extent, a program which is merely a rich experience program is not enough. *There must be respect for the new idea, freedom for planned and unplanned experimentation, and an environment filled with the excitement of discovery. Most important, there must be a guiding person who knows where the learner is, where she hopes he will go, and how he is to get there—a person who will not be hesitant in manipulating the children's environment in any way she feels will make the learning more efficient.* Only with the most careful planning and observation and with the most astute teaching, does the rich experience program become a flux that causes the learner to discover what he needs to know.

Since discovery should not be haphazard, a different way of planning the learning situation is required. Teaching is not telling. Teaching is not telling because learning is not memorizing and repeating. Wherever learning is recognized to be discovering, the teacher will set the stage for the learner so he will discover what is important for him to discover, and will do so in a way that is time-saving, direct, and most meaningful for his efficiency in society, both at the moment and as he grows older.

The Strategic Moment

If the child's search is to operate within the curriculum, the teacher must know exactly what concepts she wants to teach and how these develop in the child, and she must recognize the strategic moment when one comes.

After that, the teacher needs to learn how to stimulate the child to discovery and invention. There is no "hands-off" attitude here. Nor is it a situation in which one can teach the book or the unit. The child is always ready to be stimulated at his own developmental level.

In each of the following instances of readiness for growth in some mathematical concept, the child's need could be met best by the teacher who, herself, was "ready." She had to be ready to see, to act from an understanding of the concepts to be developed and the children developing them, and to relate these concepts efficiently to the body of knowledge the child must master.

For example, there's Billy and the stars. Billy is experimentally testing what he knows about distance out from his world into infinity. He is studying distance and degree of distance, without any numbers at all, by evolving his own highly individual "number scale" for distance in a specific direction. He has begun to establish limits to that scale, though they are implied rather than stated. One limit is himself or the ground; the height of a star is the other. He is establishing his own markers along the scale—from "houses" to "trees" to "airplanes," and then to a "maybe" that he wants to know more about.

Could it kill any of the wonder in Billy's search if the teacher at this point brought into his realm of experience literature, pictures, poetry and, yes, facts, cold hard facts, even though they might not agree with the TV-comicbook world of phony space fiction? Facts, that is, that never seemed to give the answers, but only pointed up the immensity and validity of the question Billy was asking? Yes, it could kill the wonder. It could, if the teacher presented too great, too "pat," too certain a body of learning to very young Billy. But it would not kill it if all this was presented with restraint, with its main objective the respectful recognition that this is a mystery not yet solved, a problem not yet answered, a valid area for wonder, a field in which men, like Billy, are working and questioning, trying to push the known out into the unknown, marker by marker, point by point, measurement by measurement.

Far too often, education does not begin with a problem. We start teaching, not the wonder, not the question, but answers and methods to find answers. Yet it is altogether feasible to go at it the other way 'round and so to fill the child with interest and question in all the areas of mathematics that he cannot rest till he has found the tools and skill with which to make them his own.

This wonder, this excitement, this question in the minds of all the Billies is the most important "readiness" a teacher can foster. To pick it up gently, to nurture it carefully, to stimulate it respectfully, and never to kill it with the pat and total answer—that is the greatest responsibility the teacher has. And, in the instance of Billy, this is what the strategic moment for teaching asks.

With the very youngest, as with older children, the teacher's part at a given moment may be merely to help the child clarify a term for a mathematical relationship of which he is already clearly aware. Such a need was Stephen's, at the age of three years.

STEPHEN: My birthday was tomorrow.
TEACHER: Is it tomorrow?
STEPHEN: Yes, it was.
JANET (aged 4 years, 7 months): Stevie, if you mean the day before today, it was yesterday.
STEPHEN: Yes, I know. It was yesterday.
TEACHER: Has the birthday already been?
STEPHEN: Yes, and the fudge has already been, too.

Equipment Helps

Other needs are met by careful choice of equipment which permits arithmetic learning to proceed efficiently. In the following example, two five-year-olds discover multiplication. They do it in order that they

may continue to add, and do it by a process that parallels the process of division. Without the multiple-series blocks, they might not have discovered this so soon; with them they were enabled to recognize a mathematical relationship and to define accurately the second discovery—that of the inverse relationship between the multiplication and division processes.

JON: I don't have any more long blocks. (Looks at 3-row block building.)
BOBBY: Oh, gee whiz! No more.
NORMIE: Now what? It *is* a big one. How could we build it higher?
BOBBY: (looks, pauses, looks at blocks): Oh, oh, I know. Look, do it this way. (Places two 9-inch blocks end to end.) See, this one and this one, and you have a long one. See?
NORMIE: Boy! Now we can go on along. (Brings 9-inch and 4½-inch blocks. Jon brings 4½-inch block, puts it beside 9-inch block, looks at it, stands, turns quickly, gets another 4½-inch block and places it to fill the space. Children complete building to seven rows of blocks.)

Only a week earlier these same boys had been working with this same problem, but their multiplication was without any accurate verbalization, and there was far less precision in carrying out their activity. Yet in their vagueness were indications of the problems that held their interest, and the important task for the teacher at the moment was to supply equipment and opportunity for the search to continue while she herself remained in the background.

JON: Oh, boy! Let's. Let's make a big, big, big, big, big, big building!
BOBBY: Oh, yeah! A great big, big, big, big, big, big one! You know how many it will take? It will take 150 blocks!
JON: Whew!

Number in the Program

That a number word occurred in this discussion was due not to an exact understanding of its meaning, but rather that at this time they were using it as a group term for a limit, a "whole lot," "the most." Later this number term, too, came to be ranged along a scale of increasingly exact meaning, in a way that showed a high degree of awareness of other important arithmetical concepts—that of seried relationship in amount and the more involved concept of relative values of different units.

BOBBY: He's going to give me a lollipop. A lot of lollipops. 150!—Whew! How many lollipops are you going to give me?
DICKIE: 100.
BOBBY: (stops working, looks at Dickie) Just that many?
DICKIE: They're not going to be lollipops tho'. They're something else.
BOBBY: Oh. (Returns to work, smiling.)

At this point certainly these 4- and 5-year olds are ready for working with a number system, ready for directly planned teaching about quantity. But if her decision at the moment is to begin teaching the number system, in this instance the teacher's greatest task is to be ready to teach the number system, too, *in the way the children use it and develop it*. If children's behavior is examined in order to determine how to teach the number system and whether to teach addition first, or addition and subtraction together, it is quickly seen that children work with *increase* and *decrease* at the same time. It is seen, too, that they expect, in the natural course of events, that any number series might be reversible. That is the way, in their world, things happen. And most noticeable of all is that they work with a whole body of knowledge of degrees of difference, of variation of unit value, and of concomitant variation. It may seem most logical to the teacher to present the number system as a way of working with groups and subgroups rather than simply as adding 1's. Here her own understanding of the number series will determine her success or failure, and so deserves her attention.

(Bill, Drew, Jim ride around walk.)
BILL: One is less than three. I went three times. Three times this morning.
DREW: I went, too, with you.
BILL: A hundred times is more than three. I didn't go that many.
DREW: Whew! No.

With or without number, children work with degree of quantity in this same manner.

(Stan is pushing Dick at the swings.)
STAN: How high do you want to go? (Hopefully) Not very high.
DICK: I will go high, maybe.

STAN: High, high, high. Ha, high, high.
DICK: I don't want to go *that* high.
STAN: All right. I will push you not quite that high when I get you up there.

Interrelatedness of Mathematical Concepts

With this awareness of the involved relationships the children recognize and use, a teacher must choose carefully whether she should use the moment to stimulate toward experimentation with a system of number, or instead toward increased recognition of the interrelationships the child is perceiving. Certainly, since the interest is evident in both, she should guard against concentrating on one aspect to the detriment of the other.

In the following instance nothing was told to the child but the child was stimulated to discover a specific area of differences. Through his discovery of the numberless possibilities or series of degrees of difference, a whole new field of experimentation and observation was open to him.

The teacher was told by a 5-year old that he had a dog.

CHILD: He's a St. Bernard.
TEACHER: How big is your dog?
CHILD: He's as big as every St. Bernard dog.
TEACHER: Are all St. Bernards the same size?
CHILD: Yes. Well, they're big.
TEACHER: Very big!
CHILD: Yes! I guess this big. (Shows with arms.)
TEACHER: Is every St. Bernard exactly that big?
CHILD: No! (Laughter) They're somewhere about that big; it depends.
TEACHER: On what?
CHILD: If he's a baby, he's little. Or if he's a big St. Bernard or not.
(Comment from a child standing nearby): Or if it's hot and he's had his hair all cut off.
(At this moment there was excitement evident in the behavior of the first boy.)
CHILD: He couldn't be the same size! What if he had *just one hair out*, he'd be different!

Whether moving out into space in quest of the greatest distances or in toward minute experimentation in search of the smallest difference, the child learns best when he is helped by a teacher who sees and uses the strategic moment. And as the teacher frees herself to do this, she embarks along with him on a voyage of discovery that recreates for her the excitement of all she ever felt that teaching should be.

EDITOR'S NOTE. Ah, to be a four-year-old again with understanding and imaginative parents and teachers. However, since we cannot turn back the clock, we can seek to understand how the young mind works and how it is continually reaching out with ideas and seeking refinements and "definitions" of the myriad of things that touch the young mind. It is our job as teachers of people of all ages to provide some of the media for understanding and some of the modes for reaching learning. But we must always remember that real learning is an individual matter, it is a path that each must travel and we the teachers are privileged to provide encouragement, to point out some of the milestones, and to give assistance when it is most opportune. But is it not learning when we provide a Cadillac with a chauffeur. Some people never lose that spirit of adventure that seems to be a part of childhood while others apparently lose it in the elementary school. Learning is really a glorious adventure with infinite possibility for discovery and many potential pitfalls. The role of the teacher is that of counsellor and guide. To be a good guide she must know the way and understand the child she is guiding.

Another Look at Problem Solving

Dorothy Berglund, Margaret Boyd, Elizabeth Cocke, Camilla Durrenberger, and Eunice Rader
Georgetown Elementary School, Georgetown, Texas

As a result of an inservice education program, the authors of this report undertook an extensive investigation of ways in which pupils' work with story problems could be improved. At the outset, it was recognized that this was an area of arithmetic instruction about which many teachers were concerned. After a thorough review of literature, including research articles as well as other reports, the authors hypothesized that "experience problems" would provide, at least, a partial answer to this problem. Experience problems were defined as those problems which provide a quantitative experience outside the regular textbook. In five classrooms in grades four, five, and six, extensive experience problem projects were undertaken.

Four specific steps are discussed so that the reader might understand the procedures used. These steps include: (1) compiling a file of problems; (2) use of the file; (3) use of the notebook; and (4) sharing of experiences.

(1) Each of the authors compiled a file of experience problems. Teachers, children, or teachers and children working together developed each of the experience problem statements which were written on 3×5 cards and added to the file. Each experience problem implied an activity, a familiarity with proper tools, and an understanding of certain problem relationships derived from the use of the tools. Problems were selected so that a solution provided experiences in measurement based on height, weight, time, scale, perimeter, area, volume, and acre. Experiences in the use of averages and graphs were also implied in the statement of many of the experience problems. Three samples from the files of these teachers are shown below:

1. What is the difference between the length of the jump rope and the bat?
2. What is the average weight of five of your friends in the class?
3. What is the volume of air in the classroom? What is the volume for each pupil?

(2) The file of experience problems has been used in various ways. Each teacher has permitted her class to experience the problems in the way most suitable to her room situation, which resulted in each teacher using different approaches. Certain days were designated by some as times for selecting and solving problems by the entire class, while others let only a few work with the experience problems during the time other pupils were working in their textbooks. Another plan was to let the children who completed their assignment early work in pairs with these experience problems. Too, the experience problem has been used as a forerunner for similar problems in the text. For other teachers, the experience problem replaced the textbook story problem.

(3) Each child kept his work in an individual notebook. After selecting the problem he wished to work, he copied it onto a fresh page of his notebook, returned the card to the card file, and proceeded to solve the problem, which was figured and explained in the notebook. This explanation included a discussion of the method and the tools used. Then a concluding statement was given, stating the results and facts discovered. A vocabulary of new words and arithmetical terms was listed and added to as the occasion arose.

In some instances facts and numbers were discovered by groups when carrying out activities of the day. These were used to construct experience problems that were included in the notebook.

(4) The basic purpose of sharing experiences was to facilitate the development of truly meaningful and functional concepts. Words and ideas were confused commonly in that the child thought that if he knew words, he understood the ideas they symbolized. The fact that a child, after sufficient repetition, can say that there were three feet in a yard does not guarantee that he has a true conception either of a foot or of a yard.

In their daily planning, the teachers involved in this study provided various methods and situations for the sharing of experiences that grew out of the arithmetic activities.

Since the practice of writing the experience problems constituted a variety of rich experiences, notebooks were frequently exchanged, and the problems were read and discussed. Often the tools and measures used were again discussed with the entire group or the class.

Each child was given some opportunity to share his experiences with the entire group, with a small group, with a classmate, or with the teacher. In doing so, he gave some evidence that he knew which tools or measures to select, that he was improving his concept of the use of these measures, and that he understood the basic elements of the experience problem.

The learning experiences in the modern elementary school are organized around major problem areas where subject matter boundaries are ignored, and children can move into any area in which they meet problem situations. The science and social studies fields provided activities for sharing experience problems.

These examples have been but a few of the many which could be found in any classroom. Whatever the theme of the unit, the alert teacher can find countless opportunities for building a rich background of socially significant mathematical experiences—all in the scope of a broadly conceived arithmetic program.

As a result of four months' work on the part of the authors, the group has concluded that experience problems are:

1. A challenge to pupils
2. A means of arousing interest
3. A means of broadening reading
4. A source of meaningful activity
5. A means for developing skills in writing meaningful and exact statements
6. A proper substitute for many textbook stated problems
7. A supplement for regular arithmetic activities
8. An experience which makes other stated problems more meaningful
9. An aid in discovering a child's strength or weakness
10. An aid in the child's discovering his own abilities or needs
11. A means of correlating the subject matter areas of the grade level
12. Avenues for sharing knowledges and skills.

Pupil growth was noted in many areas. The following list is indicative of specific areas in which growth has taken place. As a group, the teachers concluded that as a result of participation in an experience problem program, many of the elementary pupils:

1. Have an improved concept of the meaning of various measures
2. Have an improved knowledge of mathematical terms
3. Realize there are opportunities for use of arithmetic in all fields
4. Recognize the need for and the use of proper measuring tools
5. Have improved their skill in the use of proper measuring tools
6. Have broadened their interest and understanding of stated problems
7. Have an idea of the essentials of story problems and can better sense erroneous, absurd, or incomplete statements
8. Have become better able to associate textbook problems with life experience
9. Better evaluate details in their own experience problems and in those written by others
10. Have improved in their ability to put into words what they have actually experienced
11. Have improved their ability to estimate in a variety of situations
12. Have improved their comprehension of the relationship between classroom experience and their out-of-school life.

EDITOR'S NOTE. Here is another example to illustrate that it is desirable for a child to be an active participant in all stages of learning. The factor of interest alone is worth a great deal. And the opportunities to see arithmetic at work in various areas and to write competently add to the values in the project. Textbooks, by their very nature, cannot supply the real things with which children ought to be concerned. Let us complement the textbook with experience in which each child is a real participant.

"'Twas the Night Before Christmas"

(In the First Grade)

Dorothy S. Ambrosius
Capron, Ill.

"'Twas the night before Christmas . . ." (100% attention is what I had from my small enraptured listeners.)

"When, what to my wondering eyes should appear,

But a miniature sleigh, and *eight* tiny reindeer." (I must admit I put a little extra emphasis on the "eight" as I read it.)

"And then in a twinkling I heard on the roof,

The prancing and pawing of each little hoof."

Here I hesitated for a moment and said "My goodness, I wonder how many little hoofs that was altogether?" Then we finished the poem and said no more about it for the moment.

Later on in the day when we were doing arithmetic I asked, "Does anyone remember how many reindeer Santa Claus had?" Practically all of them knew. (Again, I had 100% attention.)

I remarked "My, what a clatter that must have been on the roof with all those tiny hoofs."

Paul wanted to know "How many were there?" Several children tried guessing.

Alan guessed "eight."

I asked "Alan, how many reindeer were there?" He answered correctly "eight."

"And how many feet does each reindeer have?"

Alan thought a moment and said "four."

"All right, Alan, come up and show me with these blocks the feet of one of Santa's reindeer."

Alan came up and took four blocks out of the box and put them two and two on my desk. I let all the children see them and we pretended that they were the feet of the reindeer. Then we divided ourselves into three groups of eight children each. Each group had a box of blocks and the problem was to find how many tiny hoofs were on the roof.

As I circulated from group to group it was enlightening to hear their suggestions on how to go about finding out. Some didn't wait to discuss but started laying out the blocks. Bradley put two blocks down and said "There's the deer's front feet" and, putting down two more he said "and there are his back feet." Ann pushed them rather close together and said "That's a whole deer. Now let's make another." I could see that these children were on the right track so I praised them and went on.

The next group of children were having an argument. They had laid out eight blocks which were to represent the reindeer but couldn't agree as to what should be done next.

"Now what are we trying to find?" I asked. "We already known how many reindeer Santa had."

They agreed that it was how many hoofs. I led them to lay out the one group of four that showed one reindeer's hoofs. Then Sandra said "We have to do that for all the reindeer." They were launched so I left them.

Just how the third group of children did it I will never know because when I got to their corner the eight block groups of four each were lined up and Tommy was just finishing counting them by "ones." The children were all eager to tell me that there were 32 hoofs.

Tommy grasps arithmetic very readily and

had previously discovered counting by 2's, so I suggested "Tommy, can you think of a faster way to count them?" Immediately he said "Yes, by 'twos'." He and the rest of the group did this successfully.

I asked them if they remembered how we arranged groups of more than nine for counting. They all knew that ten was the answer and set about arranging their groups in "tens."

Cheryl's group was the first to finish so I let Cheryl count the blocks by tens. Pointing to each group she counted "Ten, twenty, thirty, forty."

What was the matter? We all knew there were just thirty-two. There was dead silence for a moment and then a general stir as each group tried counting its blocks by "tens."

"Well," said Tommy, "you can't count by tens when there are only two blocks." (He was pointing to the last two.) Tommy counted them "Ten, twenty, thirty" and when he came to the last two he pointed to them separately and said "thirty-one, thirty-two."

All groups were successful. In a class discussion the children generalized that they were able to find many hoofs altogether by counting by "ones," by "twos," and by "tens," but that it was faster to count by "tens." I asked "How many were there altogether when we had eight groups of four blocks each?" They were all anxious to give the answer "thirty-two."

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BEN A. SUELTZ, *Editor*
State University Teachers College, Cortland, New York

AUTHOR INDEX—1958

TITLE INDEX—1958

Report of the Nominating Committee

The Committee on Nominations and Elections presents its slate of nominees for offices to be filled in the 1959 election. The term of office for the two vice-presidents is two years. Three directors are to be elected for terms of three years.

In making nominations for the three director positions, the Committee followed the directive adopted by the Board of Directors in 1955 which states, "Nominations shall be made so that there shall be not more than one director elected from each state, and that there shall be one director, and not more than two, elected from each region." Members may consult *The Mathematics Teacher* for October, 1955, for a map of the regions as they are now defined.

Ballots will be mailed on or before February 10, 1959 from the Washington Office to members of record as of that date. Ballots returned and postmarked not later than March 10, 1959 will be counted.

The committee wishes to thank the many members of NCTM for their help in giving their suggestions for nominees. It is hoped that all members of our organization will exercise their privilege of voting.

MILTON BECKMANN, *Chairman*
CLIFFORD BELL
CHARLES BUTLER
ROBERT FOUCH
MARTHA HILDEBRANDT
MILDRED KEIFFER
ANN PETERS
MYRON ROSSKOPF
MARIE WILCOX

The following people have accepted nominations:

Oscar F. Schaaf, Director from the Far West
Christine Poindexter, Director from the South West
Carol V. McCammon, Director from the South East
Henry Van Engen, Director from the North Central
Irene Sauble, Director from the North Central
Phillip Peak, Director from the Central
Z. L. Loflin, Vice-President of College Section
Phillip S. Jones, Vice-President of College Section
Marian C. Cliffe, Vice-President of the Junior High School Section
Mildred B. Cole, Vice-President of the Junior High School Section

M